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A note on Equilibrium with Capital Goods, Storage and Production

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# A NOTE ON EQUILIBRIUM WITH CAPITAL GOODS, STORAGE AND PRODUCTION\*

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## Abstract

In this paper, we study a two-period economy with production, capital goods and storage. For this economy, we propose a suitable equilibrium concept, and we prove existence of equilibrium. We also show that equality of rates of return for stored capital goods endogenously emerges as a consequence of consumers' optimizing behavior.

KEYWORDS: Capital goods, storage, existence, equality of rates of return. JEL CLASSIFICATION: C62, D24.

# 1. INTRODUCTION

In this paper, we study a two-period economy with storage and production. Services of capital goods owned by consumers are required as inputs in the production activity. Consumers trade in capital goods so as to maximize the return from their investment. In particular, they can either sell or buy capital goods on currents markets, and sell their services to the production sector using appropriate future markets.

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However, since capital goods do not provide direct utility for consumers, we can face a situation in which they are actually seen as perfect substitutes. In this case, if one of them guarantees a higher return than others, demand for it will tend to increase, while the others would not be demanded at all. Moreover, as rates of return vary, it would not be possible to represent capital goods' demand as continuous function, but only as an upper-semi-continuous correspondence.

Since this demand configuration may not be compatible with equilibrium on capital goods' market, a condition of uniformity of rates of return should be exogenously imposed on the equilibrium definition for this concept to be well defined. However, in this case the demand for capital goods becomes completely indeterminate.

In this context, if we add the assumption of constant return-to-scale technology, the demand for both capital goods and their services will be completely indeterminate. While analytically tractable, this situation appears a rather unsatisfactory description of the market for capital goods.

Therefore in this paper we try amend this situation by overcoming the indeterminacy of consumers' demand for capital goods. To do this, we assume that the latter have to be stored from one period to the other using a storage technology which can be different across both capital goods and consumers. As we will show, under reasonable assumptions on this storage technology, it is possible to obtain well-behaved and diversified demand functions for capital goods, which can therefore be incorporated in the equilibrium demand system. Moreover, a condition of equality of rates of return for stored capital goods will endogenously emerge.

For the economy to be described in detail in the following section, we propose a definition of competitive intertemporal equilibrium under the assumption that markets open in the first period only. Moreover, exploiting the fact that demand for capital goods can be incorporated directly into the overall consumers' demand system, we prove existence of equilibrium following the original approach of Todd (1979).

This paper therefore contributes - hopefully with some innovative feature - to the line of research on the walrasian model of capital accumulation, which has been studied, among others, by Diewert (1977), Garegnani (1990), Impicciatore and Rossi (1982), Montgomery (1971) and Morishima (1977).

The paper is organized as follows: in section 2 the economy is described, in section 3 the storage technology is illustrated, in section 4 the consumers' problem is analyzed, in section 5 the production sector is briefly introduced.

In section 6 we propose a suitable definition of equilibrium and we study its existence, and finally in section 7 the endogenous equalization of rates of return is discussed. The Appendix contains an illustration of the original approach of Todd (1979), which is employed in the existence proof.

# 2. The economy

The economy is populated by a finite number of consumers and firms, and lasts two periods, t = 1, 2. Consumers are denoted by h = 1, 2, ..., H. There exist L consumption goods denoted by l = 1, 2, ..., L and M capital goods denoted by m = 1, 2, ..., M. Both L and M are finite. Consumers are characterized by a consumption set  $\mathbb{X}^h \subseteq \mathbb{R}^{2L}_+$  on which a continuous utility function  $u^h : \mathbb{X}^h \to \mathbb{R}$  is defined. For each h, the utility function is assumed to satisfy local non-satiation, strict quasi-concavity and, when necessary, twice differentiability. Consumers have initial endowments of both consumption and capital goods. The former are equal to  $\bar{x}_1^h \in \mathbb{R}_{++}^L$  in period t = 1, and to zero in period t = 2. The latter are equal to  $\bar{k}^h \in \mathbb{R}_{++}^M$  in period t = 1, and to zero in period 2.

While consumption goods can be directly consumed, capital goods can only be used as inputs in the production process. By assumption, production takes place in the second period only, using as necessary inputs the services of capital goods. Since there are no initial endowments of capital goods in the second period, capital goods from the first period have to be used.

To transfer each capital good from one period to another, the use of an appropriate storage technology is required, otherwise the good is assumed to go all rotten. We assume that this technology is specific to both capital goods and consumers. Moreover, we assume that it is subject to decreasing returns, as well as to saturation, as described in what follows.

In this economy, markets open in the first period only. Through these markets, consumers and firms make choices so as to maximize their objective functions, taking all relevant prices as given. Decisions relative to the first period result in obligations to be immediately fulfilled, while those relative to the second period result in obligations to be fulfilled in t = 2.

An important characteristic of our model is that, given sale and rental prices for capital goods, the relative profitability from trading these goods is influenced by the storage technology, which we recall is consumer-specific. Therefore, it may well be possible that one consumer finds it more profitable to sell one type of capital good in the first period, while another finds it more profitable to store it for one period, and rent it in the future. Of course, since capital goods are only used a source of income, all stored capital goods will be rented in the second period, at the end of which they will completely depreciate as a consequence of their use in the production process.

To sum up, there are in total four types of markets: markets for consumption goods both in the first and in the second period, markets for capital goods in the first periods, and markets for (services of) capital goods in the second periods. Therefore, we let  $p_t = (p_{1t}, p_{2t}, ..., p_{Lt})$  denote the prices for consumption goods in period t, with  $p = (p_1, p_2)$ . Moreover, we let  $q = (q_1, q_2, ..., q_M)$ denote current (sale) prices for capital goods, and  $v = (v_1, v_2, ..., v_M)$  denote future (rental) prices. Finally, we let  $\tilde{x}^h = (\tilde{x}_1^h, \tilde{x}_2^h)$  and  $\tilde{k}^h$  denote, respectively, the demand for consumption and capital goods for all h.

## 3. The storage technology

As mentioned above, capital goods have to be stored in period t = 1 to be used as inputs for the production process in period t = 2. Therefore we assume that there exists a storage technology which is privately owned by consumers at the beginning of period t = 1. For each consumer h, this technology is represented by the map  $\sigma^h : \mathbb{R}^M_+ \to \mathbb{R}^M_+$ , where

$$\sigma^{h}(k^{h}) := \left(\sigma^{h}_{1}(k^{h}_{1}), ..., \sigma^{h}_{m}(k^{h}_{m}), ..., \sigma^{h}_{M}(k^{h}_{M})\right) .$$

is the amount of (services of) capital goods that can be rented by consumer h in t = 2. One should notice that we are assuming that the storage technology is specific to both capital goods and consumers.

Since it is a storage technology, the map  $\sigma^h$  satisfies the following constraint:

$$\sigma^h(k^h) \leqslant k^h \,. \tag{1}$$

Moreover, we assume that it satisfies the following properties:

**Assumption S.** (STORAGE) For all h, the storage technology  $\sigma^h$  is

- (i) continuous and twice differentiable,
- (*ii*) strictly concave,
- (iii) strictly increasing up the saturation point  $\hat{k}^h$ .

One should notice that the above saturation assumption implies that there exists  $\hat{k}^h$  such that  $\sigma^h(k^h) < \sigma^h(\hat{k}^h)$  for all  $k^h > \hat{k}^h$  and for all h.

**Remark**. The assumption of decreasing returns to scale in the storage technology could be reasonably justified with the presence of factors like warehouses of fixed capacity. Since the temporal structure of the model does not allow to adjust the capacity of these fixed factors in the current period, we can avoid to formally represent them, although they are required to justify the above assumption.

# 4. Consumers' problem

Consumers choose consumptions and capital goods so as to maximize their utility. Therefore, given prices (p,q,v), each consumer h chooses  $(\tilde{x}^h, \tilde{k}^h)$  solving:

$$\begin{split} \max_{x^h, k^h} & u^h(x^h) \\ \text{s.t.} & p \cdot x^h + q \cdot k^h \leqslant p \cdot \bar{x}^h + q \cdot \bar{k}^h + v \cdot \sigma^h(k^h) \,, \\ & k^h \geqslant 0 \,, \end{split}$$

where  $\bar{x}^{h} = (\bar{x}_{1}^{h}, 0)$ .

From the above budget constraint, one should notice that income comes from three possible sources: sale of consumption and capital goods in the current period, and sale of (services of) capital goods in the future period. Total income can then be employed to buy either consumption goods in both periods, or capital goods to be stored.

**Remark**. Profits are intentionally neglected in the above budget constraint as they will be equal to zero in equilibrium because of the assumption of constantreturns-to-scale technology.

Letting  $f^h(k^h) := q \cdot (\bar{k}^h - k^h) + v \cdot \sigma^h(k^h)$ , it is possible to rewrite the above problem as follows:

$$\max_{x^{h}, k^{h}} \quad u^{h}(x^{h})$$
  
s.t. 
$$p \cdot (x^{h} - \bar{x}^{h}) \leqslant f^{h}(k^{h}),$$
$$k^{h} \ge 0.$$

Since capital goods do not enter the utility function, and given that consumers always prefer more income to less, they will choose their capital investment as to maximize  $f^h$ , no matter what their optimal demand for consumption goods may be. It follows that the above problem can be solved in two steps: first, choose  $\tilde{k}^h$  so as to maximize  $f^h(k^h)$  given (q, v). Second, choose  $x^h$  so as to maximize  $u^h$  given p and  $\tilde{k}^h$ .

#### 4.1. Demand for capital goods

In the first step of their maximization problem, taking (q, v) as given, each consumer h therefore solves:

$$\max_{k^h \ge 0} \quad v \cdot \sigma^h(k^h) - q \cdot k^h \,. \tag{4}$$

Obviously, any solution of the above problem is bounded above by  $\hat{k}^h$ . Therefore in the above problem we can replace the condition  $k^h \ge 0$  with the condition  $0 \le k^h \le \hat{k}^h$ . This implies that the choice set is compact, hence that a solution exists since the objective function is continuous. By introducing the following notation:

$$r^h(k^h) := \nabla \sigma^h(k^h) \,,$$

it is possible to characterize the solution to the above problem with the following necessary and sufficient first order conditions:

$$v \odot r^h(k^h) - q \leqslant 0, \quad k^h \ge 0, \tag{5}$$

where the symbol  $\odot$  denotes the component-wise product of two vectors. Under the assumption made so far,  $\tilde{k}^h(q, v)$  is a well-defined function and it is continuous as a consequence of the Theorem of the Maximum. Given  $\tilde{k}^h(q, v)$ , we let  $f^h(q, v) := f^h\left(\tilde{k}^h(q, v)\right)$  denote the maximal income from trading in capital goods. The map  $f^h(q, v)$  has several useful properties, summarized in the following:

**Lemma 4.1.** For  $(q, v) \in \mathbb{R}^{2M}_+$ , the map  $f^h(q, v)$  is

- (i) homogeneous of degree one in (q, v),
- (*ii*) continuous,
- (iii) bounded from above and from below.

*Proof.* (i) is verified by the following chain of equalities shows:

$$\begin{aligned} f^{h}(\lambda q, \lambda v) &= \lambda q \cdot \left(\bar{k}^{h} - \tilde{k}^{h}(\lambda q, \lambda v)\right) + \lambda v \cdot \sigma^{h} \left(\tilde{k}^{h}(\lambda q, \lambda v)\right) \\ &= \lambda q \cdot \left(\bar{k}^{h} - \tilde{k}^{h}(q, v)\right) + \lambda v \cdot \sigma^{h} \left(\tilde{k}^{h}(q, v)\right) \\ &= \lambda \left(q \cdot \left(\bar{k}^{h} - \tilde{k}^{h}(q, v)\right) + v \cdot \sigma^{h} \left(\tilde{k}^{h}(q, v)\right)\right) \\ &= \lambda f^{h}(q, v); \end{aligned}$$

(*ii*) follows from continuity of  $\tilde{k}^h(q, v)$ ; to prove (*iii*) we simply observe that  $\tilde{k}^h(q, v)$  satisfies  $0 \leq \tilde{k}^h(q, v) \leq \hat{k}^h$ , where the second inequality follows from assumption S.

#### 4.2. Demand for consumption goods

In the second step, given  $f^h(q, v)$  and p, each consumer h chooses  $x^h$  so as to solve:

$$\max_{x^{h}} \quad u^{h}(x^{h})$$
  
s.t.  $p \cdot (x^{h} - \bar{x}^{h}) \leq f^{h}(q, v)$ , (6a)

$$x^h \ge 0. \tag{6b}$$

For strictly positive prices, the budget set defined by (6a) and (6b) has standard properties. In particular, it is closed and bounded. Yet, since preferences are local non-satiated, we need to consider the possibility that some consumption good's price is equal to zero, hence that the budget set is not compact. However, even in this case it is possible to obtain a welldefined and continuous demand function, using standard procedures  $\hat{a}$  la Debreu (1959).

Now let  $\gamma := (p_1, q, p_2, v)$  and C := 2(L + M). Given the assumption on preferences and storage technology, the solution to the above maximization problem is:

$$\tilde{x}^h(\gamma) : \Delta \to \mathbb{R}^{2L}_{++}$$

where  $\Delta$  is the unitary simplex with boundaries, defined as follows:

$$\Delta := \left\{ \gamma \in \mathbb{R}^C_+ \mid \sum_c \gamma_c = 1 \right\} \,.$$

#### 4.3. Consumers' excess demand

From the above analysis, it follows that for prices  $\gamma$ , the consumers' excess demand is a well-defined and continuous map  $z^h$  on  $\Delta$  given by:

$$z^{h}(\gamma) := \begin{pmatrix} \tilde{x}_{1}^{h}(\gamma) - \bar{x}_{1}^{h} \\ \tilde{k}^{h}(\gamma) - \bar{k}^{h} \\ \tilde{x}_{2}^{h}(\gamma) \\ - \tilde{\sigma}^{h}(\gamma) \end{pmatrix},$$

where  $\tilde{\sigma}^h(\gamma) = \sigma^h(\tilde{k}(\gamma))$ .

# 5. PRODUCTION SECTOR

In the economy, there also exists a finite number of firms j = 1, 2, ..., Jproducing consumption goods at t = 2 using (services of) capital goods supplied by consumers. In particular, we assume that each consumption good may be produced using a single linear activity. Moreover, we assume there exists a set of free disposal activity for each good in the economy. Finally, each activity is assumed to satisfy irreversibility of the production process.

Let P = (L + C) and let B denote the  $C \times P$  matrix, with generic column denoted by  $b_p$ . Consistently with the assumptions on the production process, such a matrix can be decomposed as B = [B' | B''], where B'' is a  $C \times C$ negative identity matrix of free disposal activities, and B' is a  $C \times L$  matrix of production activities which can be further decomposed as follows:

$$B' = \begin{bmatrix} \mathbf{0}' \\ I' \\ W' \end{bmatrix},$$

where  $\mathbf{0}'$  is a  $(L+M) \times L$  matrix with all entries equal to zero, I' is a  $L \times L$  identity matrix, and W' is a  $M \times L$  matrix of non-positive input coefficients,

with generic element  $w_m^l$  denoting the quantity of the (service of) capital good *m* necessary for the production of consumption good *l*.

As it is standard in models with constant returns to scale technology, in what follows we shall assume without loss of generality that J = 1.

Profits from activity p = 1, ..., P are equal to  $\gamma \cdot b_p$ . Given  $\gamma$ , the (representative) firms selects a level of activities  $y \in \mathbb{R}^P_+$  so as to maximize profits. Since the solution of the firm's problem is not well defined if  $\gamma \cdot b_p > 0$  for some p, only those cases for which  $\gamma \cdot b_p \leq 0$  for all p are admissible. From profit maximization it follows that  $\gamma \cdot b_p < 0$  implies  $y_p = 0$ , since no activity that make losses will be activated. Therefore, if we let  $\mathcal{B}(\gamma) := \{b_p : \gamma \cdot b_p = 0\}$ , the supply set is given by:

$$z^{s}(\gamma) := \left\{ \sum_{b_{p} \in \mathcal{B}(\gamma)} b_{p} y_{p} : y_{p} \ge 0 \right\} \,.$$

# 6. Equilibrium

At a competitive intertemporal equilibrium, all agents maximize their objective functions and markets clear. From consumers' excess demands we get the following aggregate excess demand function:

$$z(\gamma) = \sum_{h} z^{h}(\gamma) \,,$$

which is well defined and continuous on  $\Delta$ . Moreover, since in the consumers' problem the budget set holds with equality, we have that  $\pi \cdot z^h(\pi) = 0$ , hence  $\pi \cdot \sum_h z^h(\pi) = \pi \cdot z(\pi) = 0$ . This is the version of the Walras' law suitable for the present economy, in which the production sector behavior is defined only in case of non-positive profits.

Equality of supply and demand is guaranteed if there exists  $y \in \mathbb{R}^{P}_{+}$  such that  $z(\gamma) = By$ . The following definition therefore applies:

**Definition 6.1.** A competitive intertemporal equilibrium is  $(\gamma^*, y^*) \in \Delta \times \mathbb{R}^P_+$ such that  $z(\gamma^*) = By^*$  and  $\gamma^* B \leq 0$ .

**Remark**. Because of Walras' law,  $\gamma^* B y^* = 0$ , so that any activity for which  $y_p^* > 0$  makes zero profits.

Existence of an equilibrium then follows from a well known result due to Todd (1979):

#### **Proposition 1.** A competitive intertemporal equilibrium exists.

*Proof.* While referring to the original source for a complete characterization of this result, we sketch in the Appendix the main steps of the proof for ease of the reader.  $\Box$ 

#### 7. Equality of rates of return

For all capital goods such that  $q_m, v_m > 0$  in equilibrium, the following quantity is well defined and gives a measures the (marginal) rate of return to consumer h from the investment in capital good m:

$$\frac{v_m r_m^h(\tilde{k}_m^h)}{q_m} \,.$$

It is easy to see that all *stored* capital goods, i.e. all capital goods such that  $\tilde{k}_m^h > 0$ , pay off the same (marginal) rate of return to each consumer. Indeed, from (5) we get:

$$\frac{v_m r_m^h(\vec{k}_m^h)}{q_m} = \frac{v_{m'} r_{m'}^h(\vec{k}_{m'}^h)}{q_{m'}}, \qquad (7)$$

for all h and all m, m'. Moreover, all stored capital goods have the same technological return across consumers:

$$r^{h}(\tilde{k}^{h}) = r^{h'}(\tilde{k}^{h'}), \qquad (8)$$

for every m and all h, h'. It should be clear that (7) implies that all stored capital goods are (locally) perfect substitutes. However, this condition is not imposed on the equilibrium definition, but emerges endogenously as a consequence of consumers' maximizing behavior.

We also notice that the rate of return on stored capital is higher than imputed return to capital not demanded at equilibrium. Indeed, for  $\tilde{k}_m^h > \tilde{k}_{m'}^h = 0$ , (5) implies:

$$\frac{v_m r_m^h(\vec{k}_m^h)}{q_m} > \frac{v_{m'} r_{m'}^h(\vec{k}_{m'}^h)}{q_{m'}} \,. \tag{9}$$

Given differences in the storage technology across consumers, we also notice that it may be possible that different consumers choose to store different levels of the same capital good. In particular, it may be possible that some consumers do store and some others do not at all. Indeed, for  $q_m, v_m > 0$ , if the storage technologies of consumers h and h' satisfy:

$$v_m r_m^h(0) < q < v_m r_m^{h'}(0)$$
,

then we will have that  $\tilde{k}_m^{h'}>\tilde{k}_m^h=0\,.$ 

# 8. Appendix

In this appendix, we illustrate the existence proof proposed by Todd (1979), so as to better clarify the content of proposition 1. We first of all define the following sets:

$$\Gamma := \{ \gamma \in \Delta \, | \, \gamma B \leqslant 0 \} \quad \text{and} \quad T := \left\{ \tau \in \mathbb{R}^C \, \left| \, \sum_c \tau_c = 1 \right\} \right.$$

Any equilibrium price vector belong to  $\Gamma$ . The set T is the affine hull of  $\Gamma$ .<sup>1</sup> To better see this, we notice that that  $\tau \in T$  implies

$$\tau = \sum_{\gamma_i \in \Gamma} \alpha_i \gamma_i \,,$$

hence:

$$\sum_{c} \tau_{c} = \sum_{c} \sum_{i} \alpha_{i} \gamma_{ic} = \sum_{i} \alpha_{i} \sum_{c} \gamma_{ic} = \sum_{i} \alpha_{i} = 1.$$

**Remark**. If  $\Gamma$  is singleton, then  $T = \Gamma$ .

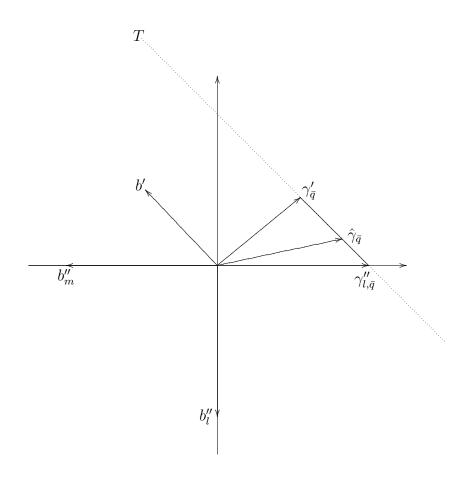
We let  $\pi : T \to \Gamma$  be the projection of T onto  $\Gamma$ . Irreversibility implies that there exists some  $\hat{\gamma} \in \Delta$  such that  $\gamma B < 0$ . Therefore, the map  $\pi$  is well defined and continuous. Geometrically,  $\pi(\tau)$  is the vector in  $\Gamma$  closest to  $\tau \in T$ , when distance is measured using the Euclidean norm. To prove the main result, define a continuous function f from  $\Gamma$  into  $\Gamma$  as follows

$$\psi(\tau) := \gamma + z(\gamma) - (\mathbf{1}_C \cdot z(\gamma)/C)\mathbf{1}_C.$$

The proof consists first in showing that if  $\tau^*$  is a fixed point of  $\psi$ , then  $\gamma^* = \pi(\tau^*)$  is an equilibrium, and subsequently in showing that indeed  $\psi$  has at least one fixed point.

<sup>&</sup>lt;sup>1</sup>Given a set X, the affine hull of X is  $\{y \mid y = \sum_{x_i \in X} \alpha_i x_i, \sum_i \alpha_i = 1\}$ , where we notice that weights are not constrained to be non-negative, as it would be in a convex combinations, but they are constrained to sum to one, as opposed to the the case of linear combination.

To better clarify the argument, we propose a simple illustration. Suppose that in t = 2 there are only one consumption good and one capital good. In this case, for given  $q = \bar{q}$ , there are only two prices relevant for the production sector, say  $\gamma_{\bar{q}} = (p_l, v_m)$ . Therefore, one may consider the following figure:



In the above figure, b' is the productive activity, while  $b_l''$  and  $b_m''$  are the two disposal activities. By construction,  $\gamma_{\bar{q}}' \cdot b' = \gamma_{l,\bar{q}}'' \cdot b_l'' = 0$ , hence  $\gamma_{\bar{q}}' \cdot b_l'' < 0$  and  $\gamma_{l,\bar{q}}'' \cdot b' < 0$ . Moreover, for any  $\hat{\gamma}_{\bar{q}}$  between  $\gamma_{\bar{q}}'$  and  $\gamma_{l,\bar{q}}'', \hat{\gamma}_{\bar{q}} \cdot b_l'' < 0$  and  $\hat{\gamma}_{\bar{q}} \cdot b' < 0$ . As mentioned in the text, the existence of at least one such  $\bar{\gamma}_{\bar{q}}$  is guaranteed by the assumption of irreversibility.

With the help of the figure, it is then easy to visualize the sets involved in the proof. Indeed, while the set T is the whole dotted line, the set  $\Gamma$  is that part of T contained between  $\gamma'_{\bar{q}}$  and  $\gamma''_{l,\bar{q}}$ , represented by the solid line. It follows that for  $\tau \in T$  to the left of  $\gamma'_{\bar{q}}$  we have that  $\pi(\tau) = \gamma'_{\bar{q}}$ , while for any  $\tau \in T$  to the right of  $\gamma''_{l,\bar{q}}$  we have that  $\pi(\tau) = \gamma''_{l,\bar{q}}$ . Finally, for any  $\tau \in T \cap \Gamma$  we have that  $\pi(\tau) = \tau$ . In this latter case, we notice that there would be no production at equilibrium.

## References

Debreu, G. (1959). Theory of Value. Yale University Press: New Haven.

- Diewert, W. (1977). Walras' theory of capital formation and the existence of a temporary equilibrium. In G. Schwödiauer (Ed.), *Equilibrium and disequilibrium in economic theory*. Reidel Publishing Company.
- Garegnani, P. (1990). Quantity of capital. In J. Eatwell, M. Milgate, and P. Newmann (Eds.), *The New Palgrave: Capital Theory*. MacMillan. London.
- Impicciatore, G. and E. Rossi (1982). Teoria neowalrasiana. Cedam. Padova.
- Montgomery, W. (1971). An interpretation of walras' theory of capital as a model of economic growth. *History of Political Economy* 3, 278–297.
- Morishima, M. (1977). *Walras' economics*. Cambridge University Press. Cambridge.
- Todd, M. (1979). A note on computing equilibria in economies with activity analysis models of production. *Journal of Mathematical Economics* 6, 135–144.

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