A model of training policies in an imperfectly competitive labour market
ABSTRACT

The model developed in this paper highlights the structure of costs and benefits on which the decisions of investment in human capital by firms and workers depend under the hypothesis of imperfect labour markets. In this case, the wage after the training period remains below productivity. Several options of training policy are analysed through the model and their outcomes compared for what concerns the level of training and other outcomes. It is confirmed that a training subsidy financed by a tax on wage of trained workers does not determine the desired effects when labour market is imperfect. On the contrary, a subsidy can be effective if it is financed through profit taxation. Moreover, when workers’ union and employers bargain over wage of trained workers, a positive effect on the total number of trainees in the economy can arise.
1. Introduction

According to human capital theory training underinvestment can arise in case of imperfect competition in the labour markets and when workers are credit constrained. In both cases training policies have to be considered in order to increase the amount of training and to alleviate the consequences of underinvestment. The model developed in this paper puts in evidence the structure of costs and benefits on which the decisions of investment in human capital by firms and workers depend under the hypothesis of imperfect labour markets. Several options of training policy are analysed through the model and their outcomes compared for what concerns the level of training and other features like the distribution of costs and benefits and wage differentials between trainees and trained workforce. The second section draws a theoretical scheme of evaluation of training policies effectiveness in connection with the structural characteristics of the markets. The third section develops the model of analysis of the enterprises’ choices relative to training under the hypothesis of an imperfect labour market and in the presence of different policies and of collective wage bargaining. The last section synthesizes main results and indicates some prospects of further research.

2. Market failures and training policy options

The economic analysis distinguishes two fundamental cases of malfunctions of the markets due to which an inefficient amount of resources is invested in training, with regards to general or, at least, not strictly specific training (Stevens 1999). The first case is that in which the labour market operates in a perfectly competitive manner, the wages are equal to labour productivity and the benefits of training stimulate workers
to entirely sustain the cost of their own training. However, difficulties emerge if credit market is imperfect and a liquidity constraint prevents workers from paying for training. It follows that, at least a part of their investment cannot be achieved, nor can it be achieved by the firms under such conditions of labour markets.

In this case, a public intervention can make it possible for such an investment to take place by replacing the missing credit up to a level retained to be socially optimal. The first obvious possibility consists in a public loan to the workers on more favourable grounds than those offered to them by the market. The same result can be obtained with a training subsidy financed by a tax levied on the wage of the qualified workers. As in the case of the loan, the subsidy would make it possible to have a larger amount of income available during the training period, thereby allowing the worker to sustain the cost in exchange for a reduction (within the limit of the tax) of the net income that he will gain after the training period.

In the second case, instead, it is assumed that there is an imperfection in the operation of labour market due to which the wage after the training period remains below productivity. Therefore the benefits deriving from training to the workers are lower, and consequently their availability to spend on it is lesser. On the other hand, the firms gain a margin of profit equal to the difference between the productivity and the wage paid to the skilled worker. The problem, however, is that a part of this profit is captured by those firms which are able to employ skilled workers without having sustained the costs (see Croce 2004). In the presence of such a positive externality, public intervention can impose upon firms the realization of a certain amount of training. This way, all firms are forced to charge themselves a part of the costs and the desired quantity of qualified workers will inflow in the labour market. An alternative intervention consists of a training subsidy financed by a tax on the profit of the firms. As a consequence of this, firms are compelled to provide the socially desirable level of training. From a theoretical point of view, therefore, each public intervention is effective under certain conditions, whereas it is totally or almost totally ineffective beyond such conditions (Tab.1).

<table>
<thead>
<tr>
<th>Causes of inefficient training</th>
<th>Remedies</th>
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<tr>
<td>Imperfect credit market</td>
<td>a. Public loans&lt;br&gt;b. Subsidies financed through tax on skilled wage</td>
</tr>
<tr>
<td>Imperfect labour market</td>
<td>a. Regulation&lt;br&gt;b. Subsidies financed through tax on profits</td>
</tr>
</tbody>
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**Tab. 1 – Policies of funding of continuous training**
3. Private investments, training policies and bargaining in an imperfect labour market

3.1. Assumptions

The model presented here develops previous works by Stevens (1996, 1999), Booth and Chatterji (1998) and Booth, Francesconi and Zogoa (1999, 2002) and it is in line with non-competitive theories of workplace training which predict that training investments are shared by workers and firms and the proportion of them sponsored by each part varies depending on several assumptions (see also Acemoglu and Pischke, 1998, 1999). Unlike Stevens (1996) in this model workers don’t pay a price to the firms for training they receive but, more realistically, accept a reduction in their wage during training period.

The model lasts two periods (we omit time discount for sake of simplicity). The economy is composed of two sectors: a primary sector comprising two firms which train their employees and, after, employ them as skilled workers, having the ‘high’ level of productivity \( \nu_2 \); a secondary sector with a large number of firms competing in a perfectly competitive labour market. Labour productivity in secondary firms is fixed at the ‘low’ level \( \nu_1 \) and no training is supplied by them. Unlike Stevens (1996), in the primary sector not only training but also production occurs in the first period. Newly hired workers are trained during working-time. Training is general (transferable) as skills are valued the same by both firms in the primary sector. At the beginning of the second period trained workers enter the skilled labour market and firms compete each other to attract them. Competition for skilled labour in the primary sector is represented as in Stevens (1996). We assume that for reasons as heterogeneous mobility costs or workers’ preferences, skilled workers are not perfectly sensitive to wage differential between the two firms (Bhaskar et al. 2002). Because of this imperfect sensitivity there is some stickiness in workers’ mobility, so that even if a firm pays a wage a little below the other, it is able to retain some workers.

We also assume, for simplicity, constant scale of returns to (both skilled and unskilled) labour. Wage in the secondary sector is constant over time and equal to productivity of unskilled labour \( \nu_1 \). Instead, productivity in the primary sector is \( \nu_1 - \delta \) in the first period, where \( \delta \) (with \( \delta < \nu_1 \)) represents the output loss proportional to the given quantity of working-time – assumed to be exogenous – devoted to training, while in the second period it is \( \nu_2 > \nu_1 \). Both firms face an identical training cost function \( C(N_h) \), where \( h = i, j \) and \( C'(N_h) > 0 \), \( C''(N_h) > 0 \), \( C(0) = 0 \). \( N_i \) and \( N_j \) represent the number of workers hired and trained, respectively, by the firm \( i \) and \( j \), whereas \( N = N_i + N_j \) is their total number in the sector.

<table>
<thead>
<tr>
<th></th>
<th>Secondary sector</th>
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<tbody>
<tr>
<td></td>
<td>Productivity</td>
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<td></td>
<td>( \nu_1 )</td>
<td>( \nu_1 - \delta )</td>
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<td>Wage</td>
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<td>( \nu_1 )</td>
<td>( \nu_2 )</td>
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<tr>
<td>Period 1</td>
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<td>Period 2</td>
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Workers and firms are risk-neutral. The number of unskilled workers employed in the secondary sector is very large at the perfectly competitive two-period income $2\nu_1$. Then at the beginning of period 1 labour supply (of potential trainees) in the primary sector is infinitely elastic at a two-period income $2\nu_1$. Every worker prefers to be employed in the primary sector if he can earn at least the same total remuneration at disposal in the secondary sector.

The degree of stickiness in the workers’ mobility is measured by the function $F(w_{2i} - w_{2j})$ which gives the probability that a trained worker chooses to be employed in firm $i$, when $w_{2i}$ and $w_{2j}$ are the wages announced by the firms at the beginning of period 2. This function is assumed to have the following properties (given $w_{2i}, w_{2j} \geq \nu_1$) (see Stevens 1996 and Booth, Francesconi and Zoega 2002):

\[
F(w_{2i} - w_{2j}) = 1 - F(w_{2j} - w_{2i}) \quad F(0) = \frac{1}{2}; \\
F'(\cdot) > 0; \quad F''(\cdot) \leq 0 \text{ with } (w_{2i} - w_{2j}) > 0; \\
F(w_{2i} - w_{2j}) \to 1 \text{ as } (w_{2i} - w_{2j}) \to \bar{x} \text{ where } \bar{x} \in (0, \infty)
\]

We also assume that neither firms nor worker know at the beginning of period 1 which preferences he will have in period 2. This implies that firms cannot act as a discriminating monopsonist but pay all workers the same wage. They only know that, given wages $w_{2i}, w_{2j}$, they will choose firm $i$ with probability $F(\cdot)$ and firm $j$ with probability $1 - F(\cdot)$. Then, the expected wage of a trained worker is $E(w_2) = w_{2i}F(\cdot) + w_{2j}[1 - F(\cdot)]$ and the participation constraint to the primary sector is $w_1 + E(w_2) \geq 2\nu_1$. However, notice that, as shown in next sections, in the symmetric setting of this model firms choose an homogeneous wage ($w_{2i} = w_{2j} = w_2$) and the expected wage is reduced to $w_2$.

### 3.2. Training in an imperfect labour market

The model has to be solved by backward induction, so we first consider the firm’s choice of the wage of the second period, then we go on to training decisions made in the first period. Firm $i$ chooses the second period wage in order to maximise its profits

\[
\pi_{2i} = (v_2 - w_{2i})F(w_{2i} - w_{2j})(N_i + N_j).
\]

The first order condition is therefore

\[
(v_2 - w_{2i}^*)F'N = FN
\]
from which the optimal wage for the firm, \( w_{2i}^* \), can be derived. At this level of wage, the benefits and the costs of an infinitesimal wage rise are equalised at the margin. In the condition above the left-hand term measures the marginal benefit of a wage rise, given by the increase in the number of trained workers employed by firm \( i \) times the surplus \( (v_2 - w_{2i}^*) \) that it captures on each one of them; the right-hand term, instead, measures the increase in the payroll costs which is proportional to the total amount of employment in the firm. The following optimal wage can be derived

\[
w_{2i}^* = v_2 - \frac{F_{i}}{F'} = v_2 - k
\]

where \( k = F/F' \). The parameter \( k \) represents the firm's surplus and can be considered as a measure of the degree of monopsony power of the firm. Its value is inversely related to the workers' sensitivity to the wage differential and tends to vanish for \( F' \to \infty \). It is demonstrated (see Appendix 1) that trained workers receive the same wage from the two firms, so that \( w_{2i}^* = w_{2j}^* = w^*_2 \).

In the first period, the firm has to decide how many unskilled workers to hire and train. At this stage the firm takes into account the total amount of profits over both periods, given the wage to be paid in the second one

\[
\pi_{1+2,i} = (\nu_1 - \delta - w_{1i})N_i + (\nu_2 - w_{2i}^*)F(w_{2i}^* - w_{2j}^*)N_j + C(N_i).
\]

In solving this problem the firm must respect the workers' participation constraint given by \( 2\nu_1 \leq w_1 + w_2 \), from which the condition \( w_1 \geq 2\nu_1 - \nu_2 + k \) descends. The firm chooses the lowest wage level satisfying it, that is \( w_1^* = 2\nu_1 - \nu_2 + k \), with \( w_1^* = w_{1i}^* = w_{1j}^* \). In other words, under our hypotheses – of general training, perfect elasticity of unskilled labour supply, risk-neutrality and absence of liquidity constraint – workers accept to cut their wage in the period 1 as this gives the firm the incentive to provide training and enables them to earn the skilled wage in the subsequent period. Besides the workers, the firms too sustain a part of training costs even though skills are general. This derives from the fact that they reap some returns to training in period 2, when they expect to gain a positive surplus over skilled employment.

From the properties given above, \( F(0) = \frac{1}{2} \), the firm's profits are

\[
\pi_{1+2,i} = (\nu_2 - \nu_1 - \delta - k)N_i + \frac{1}{2}k(N_i + N_j) - C(N_i).
\]

The first order condition relative to the number of unskilled workers hired and trained by the firm is
\[ \nu_2 - \nu_1 - \frac{1}{2} k - \delta = C^*(N_i^*) \]

where \( N_i^* \) is the optimal level for the firm \( i \). The equivalence of wages paid by firms in both periods implies as well that they decide to train the same number of workers, \( N_i^* = N_j^* \). To explain this result it must be recalled that the firm faces the risk of losing trained workers in the second period. Then, the expected value of the private marginal benefit stemming from training is just \( \frac{1}{2} k \) instead of the entire value of the monopsonistic rent \( k \). The other half of this rent corresponds to the value of the externality caused by mobility of trained workers and it appears in the condition for the maximum profit with a negative sign. This externality depresses the firm’s incentive to invest in training when labour market is not perfectly competitive and lowers the number of trainees below the socially optimal level.

### 3.3. Socially optimal level of training

The social surplus when firms train their workforce amounts to the increase of production less direct and indirect training costs. In our case, where two firms with an identical cost function are considered, this can be written as

\[ S = (\nu_2 - \nu_1 - \delta)N - 2C\left(\frac{1}{2} N \right). \]

According to this function the following condition must be satisfied in order to achieve the first-best outcome

\[ \frac{\partial S}{\partial N} = (\nu_2 - \nu_1 - \delta) - C\left(\frac{1}{2} N^f \right) = 0 \]

where \( N^f \) indicates the number of trainees maximising social surplus. This result occurs when the market for skilled labour is perfectly competitive. In this case, with perfect mobility the firms pay a second period wage \( w_2^* = \nu_2 \) and make zero profits. Moreover, given the workers’ participation constraint, the first period wage is \( w_1^* = 2\nu_1 - \nu_2 \). It follows that the two-period profit function is

\[ \pi_{1+2} = (\nu_2 - \nu_1 - \delta)N_i - C(N_i) \]
so that the firm finds profitable to train a number of workers $N_i$ such that $v_2 - v_1 - \delta = C'(N_i)$. As this condition is the same as the previous one, it follows that $N_i = N_i^f = \frac{1}{2} N^f$. In a perfect labour market firms provide exactly the socially optimal level of training (see Fig. 1). Furthermore, it is worthwhile to note that training is provided by the firm but is paid entirely by the workers by means of the reduction of the wage in the training period. When the skilled wage equals productivity, workers are induced to sustain the cost of training up to the first-best level.

Fig. 1. Number of trainees in an imperfect labour market compared to the first-best level

### 3.4. Training subsidy financed by a tax on profits

When labour market is imperfect a policy maker aimed at augmenting training incidence could pay a subsidy for every trainee. The aim of this section is to verify the effectiveness of such policy when the subsidy is financed by a tax on the firms’ profits. In particular, we assume that in period 1 the firms are given a subsidy of value $\mu$ for every trainee. On the other hand, in period 2 the firms will pay a tax proportional to the rate $\tau$ imposed on the profits they make by employing skilled workers. The equivalence between subsidies and tax revenues at an aggregate level implies $(v_2 - w_2)\delta N = \mu N$ where, as stated above, $N = N_i + N_j$. The second period profit of the firm $i$ is

$$\pi_{2i} = (v_2 - w_2)(1 - \tau) F(w_{2i} - w_{2j})(N_i + N_j)$$
and the firm’s optimal wage which results by posing equal to zero its first derivative is \( \hat{w}_2 = \nu_2 - k \). This is the same as that without policy. Even in this case it can be demonstrated that the firms settle an identical wage \( \hat{w}_2 = \hat{w}_{2j} = \hat{w}_{2j} \) (see Appendix 2). The first period wage results to be \( \hat{w}_1 = 2\nu_1 - \nu_2 + k \). Moreover, by substituting \( \hat{w}_2 \) in the equivalence condition and simplifying, we can write \( k\tau = \mu \). Then the two-period profits are

\[
\pi_{1+2,j} = (\nu_2 - \nu_1 - k - \delta)N_i + \frac{1}{2} k(1 - \tau)(N_i + N_j) - C(N_i) + k\tau N_j
\]

and the optimal number of trainees for the firm is given by

\[
\nu_2 - \nu_1 - \frac{1}{2} k(1 - \tau) - \delta = C'(\hat{N}_j).
\]

As it is shown by Fig. 2, the level of \( \hat{N} \) rises when \( \tau \) increases, and reaches the first-best level \( \hat{N}_i = N_i^f \) in the limit case \( \tau = 1 \) (the same holds true for the firm \( j \)). This demonstrates that a mechanism of subsidy and tax on profits can be effective in stimulating a higher level of training investment.

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Fig. 2. Increase in the number of trainees with subsidy and tax on profits
Intuitively, this effect can be explained by the fact that the tax is proportional to total skilled workforce employed by the firm in the second period, either internally trained or poached from outside, while the subsidy is given only for trainees hired in the first period. In other words, this mechanism transfers profits from period 2 to period 1. From the firm’s point of view this is not neutral since expected profits are reduced by the quitting probability of trained workers while subsidies increase profits of the first period with certainty.

3.5. Training subsidy financed by a tax on wage of the skilled workers

The subsidy can also be financed through taxation on wages earned by the skilled workers. In this case, in the first period the firm is given the subsidy $\mu$ for each worker hired and trained and, on the other hand, a tax rate $\phi$ is levied on the wage of trained workers in the second period. This introduces a tax-wedge such that if the firm pays $w_2$, the take-home pay is $w_2(1 - \phi)$. According to that, profits of firm $i$ in period 2 are

$$\pi_{2i} = (v_2 - w_2)\left(\mu - \phi\right)\left(1 - \phi\right)w_{2i}N_i + N_j$$

and the first order condition relative to wage is

$$(v_2 - w_2)(1 - \phi)F'N = FN.$$  

The firm’s optimal wage we obtain from this expression is $w_{2i} = v_2 - \frac{k}{1 - \phi}$. As in the previous cases the firms pay the same wage (demonstration is analogous to those in Appendices 1 and 2). If $\phi > 0$, $w_{2i}$ is below the wage paid in the case with no policy. The reason is illustrated by the condition above. The left-hand term represents the net marginal benefit for the firm of an infinitesimal wage increase. Note that the reduction of the take-home pay caused by the tax weakens the ability of the firm to attract trained workers by means of a wage increase: the number of additional workers choosing the firm $i$ as an effect of such increase amounts only to $\left(1 - \phi\right)F'N$. On the contrary, the right-hand term says that any wage increase causes a rise of payroll costs, proportional to total workforce $FN$.

The take-home pay is $w_{2i}(1 - \phi) = v_2(1 - \phi) - k$ and, on the basis of the participation constraint, the first period wage is $2v_1 - v_2(1 - \phi) + k$. Moreover, the equivalence between subsidies and tax revenues at an aggregate level implies $\mu N = \left(v_2 - \frac{k}{1 - \phi}\right)\phi N$. According to that, the following two-periods profits function can be written

$$\pi_{1+2i} = \left[-v_1 + v_2(1 - \phi) - k\right]N_i + \frac{k}{2\left(1 - \phi\right)}\left(N_i + N_j\right) - C(N_i) + \left(v_2 - \frac{k}{1 - \phi}\right)\phi N_i$$
and the first order condition relative to the number of trainees results to be

$$v_2 - v_1 - \delta - k \left[ 1 + \frac{1}{1 - \varphi} \left( \varphi - \frac{1}{2} \right) \right] = C(N_i).$$

In equilibrium the number of trainees $N_i$ chosen by the firm is inversely related to the tax rate $\varphi$. In general, we have $N_i < N_i^*$ if $\varphi > 0$. Taxation reduces the take-home pay, directly by levying the rate $\varphi$, and indirectly since the wage becomes a less powerful instrument to attract trained workers, so that the firm finds less profitable to augment the wage. Furthermore, a lower wage implies a weaker incentive for the worker to finance training by cutting the first period wage. At the end, a smaller number of unskilled workers are hired and trained by the firm. This result is in line with theoretical draft anticipated in section 3 and with Stevens (1999), who maintains that a subsidy financed by a tax on wages is unable to rise the level of training in an imperfect labour market.

### 3.6. Training and bargaining over skilled workers’ wage

So far we assumed that the wage is determined by a unilateral decision of the firm. However, it is worthwhile to consider the case of a bargaining with a workers’ union in order to examine how this can affect firm’s training decisions. For what concerns the implications of the interplay between wage bargaining and training investment there are not univocal results in the theoretical literature. In Acemoglu and Pischke (1998) firms find profitable to invest in workers’ general training when unions cause a compression of the wage structure. Otherwise, the same can happen if a monopolist union determines wage and training intensity at an industry-wide level, which is the case examined by Booth, Francesconi and Zoega (1999, 2002). In fact, contrary to the firm, an industry-wide union doesn’t suffer the externality caused by the probability of loosing skilled labour after training. Nevertheless, also a firm-level union, as Booth and Chatterji (1999) demonstrate, can favour a first-best training investment by the firm. This happens because the higher skilled wage resulting from bargaining lessens the quitting probability of trained workers and increases the expected return to training for the firm. However, unlike the case of wage compression, in this case training results from a wider wage differential between skilled and unskilled labour. Finally, various cases of interplay between bargaining and training decisions are considered in Hart and Moutos (1995).

Here we assume that workers form an industry-wide union at the beginning of period 2 in order to contend with the firms for the distribution of the surplus $v_2 - w_2$. Bargaining occurs at the industry-wide level between the workers’ union and an employers’ federation. According to that an homogeneous wage is established. Bargaining follows Nash scheme. Union’s objective is to maximize the earnings of the representative worker, whose outside option is the wage $v_1$ that he can earn in the primary sector if bargaining fails. On the other hand, the employers’ federation intends to maximize the profits of firms in the period 2. The outside option for the firm in case of bargaining failure is zero profit ($\pi_2 = 0$). Let $\tilde{w}_2$ represent the bargained wage, so that the union’s payoff is $W = \tilde{w}_2 - v_1$ and the firm’s payoff is $\pi_2 = \frac{1}{2} (v_2 - \tilde{w}_2)N$. This corresponds to
the profit function of both firms since, when an homogeneous wage $\tilde{w}_2 = \tilde{w}_2i = \tilde{w}_2j$ is established at an industry-wide level, they stop competing each other by means of rising wage offers, and for the properties of the function $F()$, it is $F(\tilde{w}_2i - \tilde{w}_2j) = F(0) = \frac{1}{2}$. The bargaining problem is

$$\max_{\pi_2} B = W^\beta \pi_2^{-\beta}$$

where $\beta$ can be interpreted as the union’s bargaining power. Then the outcome can be derived from the condition $\frac{\partial B}{\partial \tilde{w}_2} = \beta \frac{\partial W}{\partial \tilde{w}_2} \pi_2 + (1 - \beta)W \frac{\partial \pi_2}{\partial \tilde{w}_2} = 0$, which gives the value $\tilde{w}_2 = v_1 + \beta(v_2 - v_1)$. As the workers anticipate this outcome, they accept a first-period wage $\tilde{w}_1 = v_1 - \beta(v_2 - v_1)$, which satisfies their participation constraint. Substitution of $\tilde{w}_1$ and $\tilde{w}_2$ in the two-periods profit function of firm $i$ gives

$$\pi_{1+2,i} = [\beta(v_2 - v_1) - \delta]N_i + \frac{1}{2}(1 - \beta)(v_2 - v_1)(N_i + N_j) - C(N_i)$$

from which the following first order condition results

$$\frac{1}{2}(v_2 - v_1)(1 + \beta) - \delta = C'\left(\tilde{N}_i\right).$$

The same outcome is obtained for the firm $j$ as both firms pay identical wage rates. To be advantageous for the workers, bargained wage $\tilde{w}_2$ must be at least equal to the wage the firms would pay without bargaining $w_2^*$. Hence bargaining occurs if union possesses enough power according to $\beta \geq \frac{v_2 - v_1 - k}{v_2 - v_1}$. When this inequality holds in strict sense, substitution of $\beta$ in the first order condition gives $\tilde{N}_i > N_i^*$ (and $\tilde{N}_j > N_j^*$). And in the limit case of $\beta = 1$, the outcome would be $\tilde{N}_i = N_i^*$ (and $\tilde{N}_j = N_j^*$), the same as in the case of a perfectly competitive labour market. Hence, when a union bargains over the wage of skilled workers, a stronger incentive to train can arise for the firm. Notice that this derives as an indirect effect from the interplay of wage bargaining and training decisions, even if the firm continues to be the only decision-maker for what regards training. The explanation of this effect is that if the union gets a wage $\tilde{w}_2 > w_2^*$, the firm looses a fraction of the surplus $k$ of the second period but, at the same time, it receives an equivalent sum through wage reduction in the first period. However, this is advantageous for the firm, as the expected value of one unit of the surplus in the second period is just $\frac{1}{2}$, as this is the probability of retaining the trained worker. According to that, the union may help to remedy the under-provision of training arising in an imperfect labour market. This result is
consistent with the evidence emerging from several recent studies based on British dataset, as in Heyes and Stuart 1998, Börheim and Booth 2003, Booth, Francesconi and Zoega 1999 and 2003, even if further research is required to identify exact causality links.

### 3.7. Wages, cost-sharing and training

In every case analysed and illustrated in Tab. 2, apart from that of imperfect labour market, the number of trainees can reach the socially optimal level. However, in the cases C and D this should request that parameters assume their extreme, and unlikely, values $\beta = 1$ and $\tau = 1$. The sum of the first and second period wages is whenever the same, equal to $2\nu_1$, while the difference between them differs. The largest wage differential between trainees and trained workers arises in a perfect labour market whereas in the case with union and bargaining the wage profile becomes steeper as the parameter $\beta$ increases. Instead, cases A and C are characterised by the narrowest wage differential. Furthermore, in case C a higher number of trainees than in A is reached whenever $\tau > 0$ and without an enlargement of the wage differential. Steepness of wage profile over time is relevant if we admit that workers are risk adverse and credit constrained.

#### Tab. 2. Synopsis of the model results under different assumptions

<table>
<thead>
<tr>
<th>Assumptions</th>
<th>Wage of the period 1</th>
<th>Wage of the period 2</th>
<th>Difference between wages of the two periods</th>
<th>First-order conditions relative to the number of trainees</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Imperfect labour market</td>
<td>$2\nu_1 - \nu_2 + k$</td>
<td>$\nu_2 - k$</td>
<td>$2(\nu_2 - \nu_1 - k)$</td>
<td>$\nu_2 - \nu_1 - \frac{1}{2}k - \delta = C\left(N_i^s\right)$</td>
</tr>
<tr>
<td>B. Perfect labour market</td>
<td>$2\nu_1 - \nu_2$</td>
<td>$\nu_2$</td>
<td>$2(\nu_2 - \nu_1)$</td>
<td>$\nu_2 - \nu_1 - \delta = C\left(N_i^f\right)$</td>
</tr>
<tr>
<td>C. With subsidy and tax on profits</td>
<td>$2\nu_1 - \nu_2 + k$</td>
<td>$\nu_2 - k$</td>
<td>$2(\nu_2 - \nu_1 - k)$</td>
<td>$\nu_2 - \nu_1 - \frac{1}{2}k(1 - \tau) - \delta = C\left(N_i^s\right)$</td>
</tr>
<tr>
<td>D. With industry-wide wage bargaining</td>
<td>$\nu_1 - \beta(\nu_2 - \nu_1)$</td>
<td>$\nu_1 + \beta(\nu_2 - \nu_1)$</td>
<td>$2\beta(\nu_2 - \nu_1)$</td>
<td>$\frac{1}{2}(\nu_2 - \nu_1)(1 + \beta) - \delta = C\left(N_i\right)$</td>
</tr>
</tbody>
</table>

A major result of the model is the forecast of cost-sharing between worker and firm. The former finances training by lowering his wage of the period 1 below his reservation level of the same period, which is given by the wage paid in the secondary sector ($\nu_1$), by an amount equal to the increase of the second period wage above his reservation level of this period ($\nu_1$). For this reason, the total earnings of the worker over the two periods are whenever equal to his participation constraint $2\nu_1$, so that he doesn't get any net benefit from training. On
the other hand, the firm equalises at margin benefits and costs of training and finds profitable to pay a sum equivalent to the increase of its profits. Under the hypothesis of a perfectly competitive labour market – in accordance with Becker (1962) – the worker bears the whole cost. Otherwise, when an imperfect labour market is considered, the worker and the firm share the costs. In period 2 the former obtains a wage $v_2 - k$, which implies a gain above the reservation level $(v_2 - k) - v_1$. Then he is ready to reduce the first period wage by the same amount. At the same time, the firm too finances training. Its investment amounts to $\frac{1}{2}k$, that is the expected value of the surplus it captures on each skilled worker employed in period 2. Then, the total investment is given by the sum of the contributions of the two parties (minus the indirect cost $\delta$) and corresponds to the value deriving from the first order condition.

4. Conclusions

The results obtained from the model can be useful to attempt a theoretical evaluation of effectiveness of training policies. Firstly, it is confirmed that a training subsidy financed by a tax on wage of trained workers does not determine the desired effects. On the contrary, a subsidy can be effective if it is financed through profit taxation. Second, our results demonstrate that workers and firms share training investments and the proportion of the costs financed by each side depends on the distribution of benefits determined by structural features of the labour market. Third, when workers’ union and employers bargain over wage of trained workers, a positive effect on the total number of trainees in the economy can arise. Yet, several basic assumptions that can limit the validity of these propositions in some respects must be recalled, as they indicate further lines of research in this field. First of all, we only considered the case of labour market imperfections without paying attention to the possibility of credit constraints preventing workers from investing in their training. Moreover, also ‘training market’ imperfections caused by substantial problems of asymmetric information between the firm which provides the training and the worker who pays for it should be considered. As in large part of the literature, our model is a static one, in the sense that it doesn’t take into account explicitly of neither technical and organisational innovations nor the “culture” of the players (employers, workers, and their respective associations). To finish, the model concentrates on incentive structure underlying training investment decisions and on its sensitivity to alternative policies, without considering a set of further institutional factors – as arrangements of working and training-time, workforce classification inside the firms, and certification – which play a major role in training systems.

References


Appendix 1

Following Stevens (1996), profit equations of the two firms in period 2 are

\[
\begin{align*}
\pi_{2i} &= (v_2 - w_{2i}) F(w_{2i} - w_{2j}) (N_i + N_j) \\
\pi_{2j} &= (v_2 - w_{2j}) \left[1 - F(w_{2i} - w_{2j})\right] (N_i + N_j)
\end{align*}
\]

Then first-order conditions are

\[
\begin{align*}
\left(v_2 - w_{2i}\right) F' N &= FN \\
\left(v_2 - w_{2j}\right) F' N &= 1 - FN
\end{align*}
\]

The solution of these gives the optimal wages \( w_{2i}^* = v_2 - \frac{F}{F'} \) and \( w_{2j}^* = v_2 - \frac{1 - F}{F'} \). If we define \( x = w_{2i} - w_{2j} \), it results \( w_{2i} - w_{2j} = \frac{-2F}{F'} \). Given the properties of \( F() \), for which \( F(0) = \frac{1}{2} \), this equation is valid only if \( x = 0 \).

Appendix 2

In this case it is possible to proceed as in the Appendix 1. The profit equations in period 2 are:

\[
\begin{align*}
\pi_{2i} &= (v_2 - w_{2i}) (1 - \tau) F(w_{2i} - w_{2j}) (N_i + N_j) \\
\pi_{2j} &= (v_2 - w_{2j}) (1 - \tau) \left[1 - F(w_{2i} - w_{2j})\right] (N_i + N_j)
\end{align*}
\]

and the first-order conditions

\[
\begin{align*}
\left(v_2 - \tilde{w}_{2i}\right) (1 - \tau) F' N &= (1 - \tau) FN \\
\left(v_2 - \tilde{w}_{2j}\right) (1 - \tau) F' N &= (1 - \tau)(1 - F) N
\end{align*}
\]

The optimal wages which solve these conditions are the same as those obtained in the case without taxation, then it follows that \( x = \tilde{w}_{2i} - \tilde{w}_{2j} = 0 \) as demonstrated above in Appendix 1.
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