WORKING PAPER
DIPARTIMENTO DI ECONOMIA PUBBLICA

Working Paper n.110

Giuseppe Ciccarone ed Enrico Marchetti

Revisiting the role of multiplicative uncertainty in a model without inflationary bias

Roma, Febbraio 2008

UNIVERSITA' DEGLI STUDI DI ROMA
"LA SAPIENZA"

Revisiting the role of multiplicative uncertainty in a model without inflationary bias

Giuseppe Ciccarone*† and Enrico Marchetti‡

* Sapienza University of Rome, Department of Public Economics

‡University of Naples "Parthenope", Department of Economic Studies

Submitted: November 2007

Abstract

Kobayashi (2003) aims to show that, in a model without inflationary bias, an increase in the degree of multiplicative uncertainty on the transmission mechanism of monetary policy improves social welfare when central bank's preferences are highly uncertain. We demonstrate that this result applies only to the case in which society is strictly conservative, i.e., when the weight attached to output in the social welfare function is lower than one.

Keywords: Multiplicative uncertainty; Brainard conservatism; Uncertain preferences; Monetary policy

JEL classification: E52; E58.

† Corresponding author: Giuseppe Ciccarone, Department of Public Economics, University of Rome "La Sapienza", Via del Castro Laurenziano 9, 00161, Rome, Italy; email: giuseppe.ciccarone@uniroma1.it. We thank an anonymous referee for useful comments and suggestions. Financial support from the Sapienza University of Rome and MIUR is gratefully acknowledged. The usual disclaimer applies.

Revisiting the role of multiplicative uncertainty in a model without inflationary bias

1. Introduction

In a recent contribution, Kobayashi (2003) sets out to show that, in a model without inflationary bias, an increase in multiplicative uncertainty in the transmission mechanism of monetary policy improves social welfare if the degree of central bank (CB) "opacity" is high enough, i.e., if the weights attached to its objective function are sufficiently uncertain. We show that this result holds only in a specific case, i.e., when society is "conservative" and provide an explanation for this result.

2. Kobayashi's (2003) model

Kobayashi's (2003) model (in logs) is made up by a private sector forming rational expectations on inflation and a CB setting the money supply in order to minimise the loss function:

$$L_{MA} = (1+\alpha)\pi^2 + (\lambda - \alpha)y^2, \qquad \lambda > 0 \tag{1}$$

where y is output, π inflation, $\alpha \in [-1, \lambda]$ is a random variable,[1] with expected value $E(\alpha) = 0$ and variance σ_α^2, which represents the degree of opacity associated with CB's preferences. The welfare loss function for society as a whole is: $L_S = \pi^2 + \lambda y^2$. It is hence $E(L_S) = E(L_{MA})$: the central banker is randomly selected from society.

Aggregate supply is given by a Lucas "surprise" function, with the natural rate of output normalized to zero:

$$y = \pi - \pi^e - \varepsilon \tag{2}$$

[1] Note that the extrema of the interval [-1; λ] represent the cases of "fully populist" and "fully conservative" CB, respectively.

where π^e is expected inflation, and ε is a supply shock with zero mean, $E[\varepsilon] = 0$, and constant variance σ_ε^2. By setting the money growth rate, m, the CB can imperfectly control inflation, due to multiplicative uncertainty in the transmission mechanism of monetary policy:

$$\pi = (1+v)m \tag{3}$$

where v is a random variable with $E(v) = 0$ and constant variance $\sigma_v{}^2$; α, v and ε are independently distributed.[2]

The game is solved by employing a standard backward procedure, so that minimisation of (1) subject to (2) and (3) gives the CB's reaction function:[3] $m = \frac{(\lambda-\alpha)(\pi^e+\varepsilon)}{(1+\lambda)(1+\sigma_v^2)}$. Since in the absence of time inconsistency problems it is $\pi^e = 0$,[4] it follows that:

$$\pi^* = (1+v)\frac{(\lambda-\alpha)}{(1+\lambda)(1+\sigma_v^2)}\varepsilon; \qquad y^* = \frac{(1+v)(\lambda-\alpha)-(1+\lambda)(1+\sigma_v{}^2)}{(1+\lambda)(1+\sigma_v^2)}\varepsilon$$

The expected social loss is hence:

$$E(L_S) = \underbrace{\frac{(\lambda^2+\sigma_\alpha^2)}{(1+\lambda)^2(1+\sigma_v^2)}\sigma_\varepsilon^2}_{=\ \sigma_\pi^2} + \underbrace{\frac{\lambda^2+\sigma_\alpha^2+(1+\lambda)\left[(1+\lambda)^2(1+\sigma_v{}^2)-2\lambda\right]}{(1+\lambda)^2(1+\sigma_v^2)}\sigma_\varepsilon^2}_{=\ \sigma_y^2} \cdot \lambda \tag{4}$$

An increase in multiplicative uncertainty ($\sigma_v{}^2$) decreases $\sigma_\pi{}^2$ (the variance of inflation), but has an ambiguous effect on $\sigma_y{}^2$ (the variance of output). By calculating the derivative:

---

[2] The presence of α raises the question of how rational agents should behave and form their expectations in repeated games, conditional on having information on λ, which is a behavioural – thus known – parameter. In the literature on CB transparency it is generally assumed that it is difficult for the private sector to disentangle the effects of preference uncertainty and other random shock (e.g., ε and v), so that also in a repeated game setting the hypothesis on the probability distribution of λ can be maintained (see, e.g., the discussion in Muscatelli 1998).

[3] The timing of the game is as follows: (i) the private sector forms rational expectations; (ii) the supply shock ε occurs; (iii) α is revealed only to the CB; (iv) the CB chooses m; (iv) v materializes, and inflation and output are obtained.

[4] This is basically due to the fact that, there being no distortions in the economy, both players share the same bliss points: $\pi^B = y^B = 0$.

$$\frac{\partial E(L_S)}{\partial \sigma_v{}^2} = \frac{\lambda^2 - \sigma_\alpha^2}{(1+\lambda)(1+\sigma_v^2)^2}\sigma_\varepsilon^2 \tag{5}$$

Kobayashi gets that an increase in $\sigma_v{}^2$ reduces social loss if $\lambda^2 < \sigma_\alpha{}^2$. If opacity is high enough, the effect of the increase in multiplicative uncertainty on the variance of inflation will dominate that on the variance of output, and Brainard (1967) conservatism principle, in the face of multiplicative uncertainty, allows to improve social welfare.

3. Revisiting Kobayashi's (2003) result

Is it possible that the condition $\lambda^2 < \sigma_\alpha{}^2$ may hold without constraints? In this section we show that, in general, it is necessary to pose some bound on the magnitude of opacity σ_α^2.

As α must take values in a compact set, it is necessary to assess the behaviour of the variance of a random variable subject to the qualifications: $\alpha \in [-1, \lambda]$ and $E(\alpha) = 0$. The general problem of characterizing the moments of a random variable subject to specific constraints (e.g., to take values in a compact set) is a well-known issue in mathematical statistics (see, e.g., Kemperman 1968) and, without entering the general geometric approach that can be employed to solve this problem, the following sketchy argument may be sufficient for our purpose (Ciccarone, Di Bartolomeo and Marchetti, 2007). The probability distribution of α ensuring the highest variance is the one that assigns positive probability values only to the extrema of α (–1 and λ) and zero elsewhere:

$$(\alpha_1 = -1) \sim p \quad \text{and} \qquad (\alpha_2 = \lambda) \sim 1 - p$$

The distribution p is subject to the following constraint on the expected value: $E(\alpha) = p(-1) + (1-p)\lambda = 0$. We can thus set the problem of finding the distribution p which maximises the variance of α:

$$\max_p \sigma_\alpha^2 = E(\alpha - 0)^2 = p + (1-p)\lambda^2$$
$$\text{s.t.} \quad E(\alpha) = (1-p)\lambda - p = 0$$

From the first order conditions it straightforwardly follows that: $p_{\max} = \dfrac{\lambda}{1+\lambda}$. The maximum value for the variance is hence:

$$\left(\sigma_\alpha^2\right)_{\max} = \frac{\lambda}{1+\lambda} + \left(1 - \frac{\lambda}{1+\lambda}\right)\lambda^2 = \lambda$$

Thus, if $\lambda > 1$, it can *never* be $\lambda^2 < {\sigma_\alpha}^2$: increases in multiplicative uncertainty always reduce expected social welfare. The opposite result may hold only if $0 < \lambda < 1$, i.e., *society* is strictly "conservative" (and the CB is "conservative on average"). In this case, the sign of expression (5) depends on the sign of its numerator, which is a continuous and monotonically decreasing function of σ_α^2; this sign may be negative for relatively high values of σ_α^2.

4. Discussion

Given the variance of the output shock, the variance of inflation σ_π^2 depends (see equation 4):

i. positively on the degree of opacity $\left(\frac{\partial \sigma_\pi^2}{\partial \sigma_\alpha^2} = \frac{\sigma_\varepsilon^2}{(1+\lambda)^2\left(1+\sigma_v^2\right)} > 0\right)$;

ii. negatively on multiplicative uncertainty $\left(\frac{\partial \sigma_\pi^2}{\partial \sigma_v^2} = \frac{-\left(\lambda^2 + \sigma_\alpha^2\right)}{(1+\lambda)^2\left(1+\sigma_v^2\right)^2}\sigma_\varepsilon^2 < 0\right)$.

The variance of output, $\sigma_y^2 = \sigma_\pi^2 + \frac{1+\lambda^2+{\sigma_v}^2(1+\lambda)^2}{(1+\lambda)\left(1+\sigma_v^2\right)}\sigma_\varepsilon^2$, which is always greater than that of inflation, increases with opacity $\left(\frac{\partial \sigma_y^2}{\partial \sigma_\alpha^2} = \frac{\partial \sigma_\pi^2}{\partial \sigma_\alpha^2} > 0\right)$ and with the variance of the multiplicative shock $\left(\frac{\partial \sigma_y^2}{\partial \sigma_v^2} = \frac{2\lambda + \lambda^2 - \sigma_\alpha^2}{(1+\lambda)^2\left(1+\sigma_v^2\right)^2}\sigma_\varepsilon^2 > 0\right)$, since it is $\lambda \geq \sigma_\alpha^2$.

Hence:

1. an increase in opacity raises the variability of both inflation and output, thus worsening social welfare;
2. an increase in σ_v^2 reduces σ_π^2 but increases σ_y^2:

2.1. if $\lambda = \sigma_\alpha^2$ (maximum opacity) the effects of an increase in σ_v^2 on σ_π^2 and on σ_y^2 are the same in absolute value: if $\lambda < 1$, $\frac{\partial E(L_S)}{\partial \sigma_v{}^2} = \frac{\partial \sigma_\pi^2}{\partial \sigma_v^2} + \lambda \frac{\partial \sigma_y^2}{\partial \sigma_v^2} < 0$; if $\lambda > 1$, $\frac{\partial E(L_S)}{\partial \sigma_v{}^2} > 0$;

2.2. if $\lambda > \sigma_\alpha^2$ an increase in σ_v^2 reduces σ_π^2 less than it increases σ_y^2; we must then consider: $\frac{\partial E(L_S)}{\partial \sigma_v{}^2} = \frac{\partial \sigma_\pi^2}{\partial \sigma_v^2} + \lambda \frac{\partial \sigma_y^2}{\partial \sigma_v^2} = \left[\frac{-\left(\lambda^2 + \sigma_\alpha^2\right)}{(1+\lambda)^2\left(1+\sigma_v^2\right)^2} + \lambda \frac{2\lambda + \lambda^2 - \sigma_\alpha^2}{(1+\lambda)^2\left(1+\sigma_v^2\right)^2} \right] \sigma_\varepsilon^2$ and compare (in absolute values) σ_α^2 with $\lambda^2 + \lambda^3 - \lambda\sigma_\alpha^2$. If $\lambda > 1$ and $\sigma_\alpha^2 = 0$ it will of course be $\lambda^2 + \lambda^3 - \lambda\sigma_\alpha^2 > \sigma_\alpha^2$. As σ_α^2 is increased above zero, the left hand side falls more than the right hand side increases, but when σ_α^2 reaches its maximum value $\lambda = \sigma_\alpha^2$ we know that $\frac{\partial \sigma_\pi^2}{\partial \sigma_v^2}$ and $\frac{\partial \sigma_y^2}{\partial \sigma_v^2}$ are the same in absolute value, so that $\lambda \frac{\partial \sigma_y^2}{\partial \sigma_v^2} > \frac{\partial \sigma_\pi^2}{\partial \sigma_v^2}$ in absolute value. Hence, if $\lambda > 1$, it is $\frac{\partial E(L_S)}{\partial \sigma_v{}^2} > 0$ because the greater volatility of output, multiplied for a weight greater than one, has a negative effect on social welfare which is always greater than the positive effect produced by the lower volatility in inflation, multiplied for a weight equal to one.

The fundamental difference between our result and Kobayashi's interpretation lies in the fact that the constraint $\lambda \geq \sigma_\alpha^2$ univocally determines the sign of $\frac{\partial \sigma_y^2}{\partial \sigma_v^2} = \frac{2\lambda + \lambda^2 - \sigma_\alpha^2}{(1+\lambda)^2\left(1+\sigma_v^2\right)^2} \sigma_\varepsilon^2 > 0$.

Private agents always set $\pi^e = 0$, thus never compensating the output shock, even when they face greater uncertainty when forecasting the CB's behaviour; thus the greater source of uncertainty associated with α leads them to make greater *ex-post* inflation forecast errors.

A more populist CB tries to "translate" the effect of the output shock more on the variability of inflation, but this does not guarantee that the volatility of output decreases (the more so the higher is opacity). This kind of behaviour is contrasted by Brainard's conservatism principle, which leads the CB to be more prudent in the use of its instrument *m*: the variability of inflation falls, as the CB is less inclined to react to the supply shock by changing the value of the monetary instrument.

If the economy is in the optimal social point, π = 0 and y = 0, and a negative shock ε (which produces an expansionary effect on y) occurs, the CB observes the shock ε and reacts by decreasing $m = \frac{(\lambda - \alpha)}{(1+\lambda)(1+\sigma_v^2)}\varepsilon$. The greater σ_v^2 the lower the reaction, the lower the fall in inflation and the greater the increase in output. Hence, an increase in σ_v^2 brings the CB's actual behaviour closer to the expectation made by the private sector that the CB will maintain π = 0.

The consequences of an increase in multiplicative uncertainty are amplified by the size of σ_α^2. The greater is opacity the greater is the (negative) effect of σ_v^2 on σ_π^2, according to $\frac{\partial\sigma_\pi^2}{\partial\sigma_v^2} = \frac{-\left(\lambda^2 + \sigma_\alpha^2\right)}{(1+\lambda)^2\left(1+\sigma_v^2\right)^2}\sigma_\varepsilon^2 < 0$, and the lower is its positive effect on σ_y^2, according to $\frac{\partial\sigma_y^2}{\partial\sigma_v^2} = \frac{2\lambda + \lambda^2 - \sigma_\alpha^2}{(1+\lambda)^2\left(1+\sigma_v^2\right)^2}\sigma_\varepsilon^2 > 0$.

If σ_v^2 increases the CB becomes more prudent and inflation variability (which is a component of output variability) falls. The greater is opacity the sharper is this effect and the lower is the increase in output variability. Both effects improve the consequences of greater multiplicative uncertainty on expected social welfare (even though the *direct* effect of greater opacity on expected welfare is negative, as it increases both σ_π^2 and σ_y^2). This is not due to any influence of greater opacity on the private sector's behaviour (its instrumental variable is always equal to zero), but simply to the fact that the greater uncertainty produced by the "wider" variability of CB's preferences magnifies the effects of multiplicative uncertainty.

When opacity is at its maximum value, $\lambda = \sigma_\alpha^2$, the effects of an increase in σ_v^2 on σ_π^2 and on σ_y^2 exactly compensate each other, and it is the weight attached to output variability (λ) that establishes the sign of $\frac{\partial E(L_S)}{\partial {\sigma_v}^2}$. The problem is hence that, with $\lambda > 1$, the statistical bound on σ_α^2 prevents this form of uncertainty from spreading in a sufficient manner its positive influence on expected welfare, something that can instead happen when the weight attached by society to output variability is lower than one.

5. Conclusion

We have shown that for Kobayashi's (2003) result to hold society (as well the CB "on average") must be conservative, i.e., in the social welfare loss it must attach to output a weight (λ) lower than one. From the technical point of view, this is due to the fact that there exists a maximum value for opacity, which can never be greater than λ. This constraint prevents an increase in multiplicative uncertainty from lowering "enough" the variability of output. From the economic point of view, the increase in multiplicative uncertainty makes the CB more prudent and inflation variability falls. The greater is opacity the sharper is this effect and the lower is the associate increase in output variability (which depends also on inflation variability). Both effects have favourable consequences on expected social welfare, but when society is populist the statistical bound on opacity prevents it from spreading in a sufficient manner its positive influence on expected welfare, something that can instead happen when λ is lower than one: when multiplicative uncertainty increases, greater opacity can be beneficial to society only when the CB is expected to be conservative.

References

Brainard, W.C., 1967. Uncertainty and the effectiveness of policy. *American Economic Review* 57, 411–425.

Ciccarone, G., Di Bartolomeo, G. and Marchetti, E., 2007. Unions, fiscal policy and central bank transparency. *The Manchester School* 75, 617-633.

Kemperman, J.H.B., 1968. The general moment problem. A geometric approach. *The Annals of Mathematical Statistics* 39, 93-122.

Kobayashi, T., 2003, Multiplicative uncertainty in a model without inflationary bias. *Economics Letters* 80, 317-321.

Muscatelli, V.A., 1998, Optimal inflation contracts and inflation targets with uncertain central bank preferences: accountability through independence? *The Economic Journal* 108, 529-542.

Pubblicato in proprio
Dipartimento di Economia Pubblica
Facoltà di Economia
Università degli Studi di Roma “La Sapienza”
Via del Castro Laurenziano 9 – 00161 Roma

Working Paper del Dipartimento di Economia Pubblica
Università degli studi di Roma "La Sapienza"
Via del Castro Laurenziano 9 – 00161 Roma

I Working Paper vengono pubblicati per favorire la tempestiva divulgazione, in forma provvisoria o definitiva, dei risultati delle ricerche sulla teoria e la politica economica. La pubblicazione dei lavori è soggetta all'approvazione del Comitato Scientifico, sentito il parere di un referee.

I Working Paper del Dipartimento di Economia Pubblica ottemperano agli obblighi previsti dall'art. 1 del D.L.: 31.8.45 n. 660 e dal Decreto del Presidente della Repubblica 3 maggio 2006 n.252, art.37.