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Linear Contracts, Common Agency and Central Bank Preference Uncertainty

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# Linear contracts, common agency and central bank preference uncertainty[1]

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**Abstratc**

The aim of this paper is to bring together two recent developments in the "contracting" approach to the time-inconsistency problem of monetary policy: linear contracts under common agency and central bank preference uncertainty under single agency. We show that under common agency and imperfect "political" transparencey, the full transparency finding that the interest group contract dominates the government's one is confirmed, but equilibrium expected inflation is lower, as the new source of uncertainty makes the two principals more cautious in their instrument setting. This reduces the average inflation bias. We then extend the analysis to the case of uncertainty on the central bank output target and show that the expected values of inflation and output are the same as those obtained under perfect "economic" transparency, whereas the actual values are different only for the presence of an additive term depending on opacity. Finally, we demonstrate that when the principals are uncertain about the weight attached by the central banker to the incentive scheme the equilibrium inflation surprise may be negative and output may be lower than the natural rate.

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# 1 Introduction

The aim of this paper is to bring together two recent developments in the so-called "contracting" approach to the time-inconsistency problem of monetary policy (Persson and Tabellini [31]; Walsh [35]; Waller [34]; Frattianni, Von Hagen and Waller [19]; Candel-Sánchez and Campoy-Miñarro[5]; Chortareas and Miller[8]). According to this principal-agent approach, it is possible to design an optimal incentive scheme able to produce monetary policy outcomes equivalent to those obtained under credible commitment. If the central banker can be conceived as (at least in part) an egoist agent, i.e., a self-interested utility maximizer whose decisions depend on private rewards (Chortareas and Miller [6]), under bilateral agency (one principal and one agent) she may be taken to respond to incentive schemes offered by her principal (society, represented by a government). The principal can hence design a formal contract with the central banker which, from the analytical viewpoint, amounts to introducing a linear incentive scheme in her loss function that penalizes deviations of actual inflation from its target. It is well known that an optimal penalty rate exists that generates expectations of money growth and inflation that are both equal to zero (Walsh [35]). This occurs because the incentive contract increases the marginal cost for the central banker of higher money growth rates, thus counteracting her inflationary bias.

The first development of this approach we shall build upon is represented by Chortareas and Miller's [7], hereafter CM, extension of the analysis of the linear contract to the common agency. CM assume that the central banker can enter into a formal contract with the government (emerging from the political process as the collective decision of society) and into an informal contract with a private interest group reflecting a relevant disaffected minority.[4] CM also convincingly discuss the motivation to introduce an interest group into the contracting approach, as this can accurately describe important features of the actual economic environment, such as the presence of different sectors, or industries, or branches of the government having different preferences between inflation and output. The issue seems particularly important in common currency areas (Dixit and Jensen [15]), especially in the European context where, after the creation of the European Central Bank, some national governments may be envisaged as interest groups lobbying, e.g., for low interest rate, as it would occur in the case of high-debt member states, or of countries experiencing low rates of output growth.

The interest group may have several different aims, but here we restrict our attention to what we reckon as the most interesting case, i.e., that in which monetary policy is delegated through inflation contracts and the second principal offers a competing output contract (i.e., the interest group offers and incentive scheme linked to output, not to inflation). The main result CM achieve is that the government contract does not deliver the commitment-equivalent results, as the output contract dominates the inflation contract in affecting the central banker's incentives: the expected money growth rate and

[4] As the latter contract is not likely to be explicitly announced, CM use a definition by Laffont and Tirole [28] and label it as quasi-enforceable, meaning that the parties are taken to willingly enforce the contract promises.

actual inflation which obtain when only the interest group offers an incentive scheme to the central banker, while the government does not, are exactly the same as those that crop up when both principals offer their contracts. Yet, output remains at the natural level. The dominating incentive scheme provided by the interest group hence reproduces the traditional discretionary outcome of Barro-Gordon [1] type models.

The second development of the contracting approach we shall consider is represented by the introduction of imperfect central bank transparency into the bilateral agency model (Muscatelli [29]; [30]; Beetsma and Jensen [2]). We rate this as a particularly relevant issue because, in spite of a rapidly growing debate, the macroeconomic effects of transparency on inflation and output (levels and variabilities), and hence on social welfare, remain controversial at both the theoretical and the empirical levels.[5] Two main types of transparency have been envisaged in the literature. The first one is related to information asymmetries between the central bank and the general public about the weight attached to the arguments of the central bank's objective function;[6] this is known in the literature as the "contingent", or "political", view of transparency. The second one refers to uncertainty on the output target of the central bank, i.e., the so-called "economic" view of transparency,[7] due, for example, to uncertainty about the NAIRU, or to fluctuations in it.

When imperfect political transparency is introduced into the bilateral agency model, the optimal solution provided by the linear contract no longer applies and a stochastic inflation bias persists in equilibrium, reproposing the trade-off between reducing the inflation bias and stabilizing supply shocks (Muscatelli [29]; [30]; Beetsma and Jensen [2]). Another important result is that under uncertainty on the central bank's output target, the optimal contract brings equilibrium inflation which is certainty-equivalent to that obtained under full "economic" transparency: expected inflation rates are the same, and actual inflation depends on the value of the output target[8] (Muscatelli [30], section I, case c).

Another type of preference uncertainty, which may be labelled "selfishness" uncertainty, crops up from the assumption that the central banker's trade-off between social welfare and the incentive scheme is private information, i.e., that the central banker's responsiveness to the incentive scheme is not perfectly known by all principals. Under this assumption, and always in the case of bilateral agency, CM [6] showed that a benevolent central banker with an inflation bias has an incentive to pretend to be selfish, and hence to accept a contract designed for a selfish central banker, because her loss is in this case

[5] See, among others, Kuttner and Posen [27]; Chortareas et al. [9]; Geraats [20]; Grüner [23]; Beetsma and Jensen [3]; Posen [32]; Di Bartolomeo and Marchetti [14]; Eijffinger and Geraats [16]; Ciccarone et al. [10]; Demertzis and Hughes- Hallett [12]; van der Cruijsen and Eijffinger [33]. See also the papers contained in the special March 2007 issue of the *European Journal of Political Economy* (volume 23) on "Central Bank Transparency and Central Bank Communication".

[6] See, among others, Faust and Svensson [17] [18], Cukierman [11] and Jensen [25].

[7] It should be noted that there is no general agreements on these labels. For example, Geraats [20] defines the uncertainty on target levels as imperfect political transparency.

[8] In this context, delegation through incentive contracts is superior to discretion only if the uncertainty on the output target is not too large.

lower than that she would experience if she rejected the contract and a truly selfish central banker were appointed to conduct monetary policy. CM [6] convincingly document that the offspring of this behavior is that positive inflation surprises occur and equilibrium output is greater than its natural rate.

In this paper we aim to identify the changes produced by the introduction of the three types of central bank preference uncertainty summarized above on the effects of linear contracts in the common agency set up. To this aim, in the next section we study the problem under the "political" notion of transparency (uncertainty refers to the weights attached to the "social" objectives, i.e., output and inflation, whereas the weight attached to the linear contract is assumed to be perfectly known by all principals). In section 2, we instead assume that there exists uncertainty only on the central bank's output target, so as to study the effects produced under common agency by the consideration of imperfect "economic" transparency. In section 3 we extend to the common agency case CM's [6] assumption that the central banker's responsiveness to the incentive scheme is not perfectly known. Section 4 concludes.

The results we obtain confirm only in part the existing findings, highlighting important differences produced by preference uncertainty in the common agency set up. Whereas the interest group contract is always dominant under the three forms of uncertainty we look at, the specific kind of uncertainty which is considered plays a crucial role as far as the effects of the linear contract under common agency are concerned. Under imperfect political transparency equilibrium expected inflation is lower that under perfect transparency, as opacity plays a "disciplining" role on pressure groups by inducing them to choose less aggressive lobbying strategies. Furthermore, there exists a value of opacity for which inflation expectations are equal to zero, but the commitment solution can be obtained only on average. As for the second type of preference uncertainty, our main finding is that economic imperfect transparency has no effect on expected values. Finally, under "selfishness" uncertainty, we do not confirm CM's [6] single agency finding that it is always convenient for a benevolent central banker to pretend to be selfish and subsequently break the incentive contracts. Contrary to what happens under single agency, when this happens, the central banker may generate a negative inflation surprise: equilibrium output may be lower than the natural rate.

## 1.1 Common agency and uncertain "political" transparency

The macroeconomic set up of the model is a standard one. The aggregate supply side of the economy is summarized in a standard AS function incorporating the surprise inflation effect:

$$y = y^n + \alpha \left(\pi - \pi^e\right) + \varepsilon \tag{1}$$

where $y$ and $y^n$ are the actual and the natural levels of output, $\pi$ is the inflation rate, $\pi^e$ is the private sector's rational expectation of the same rate and $\varepsilon$ is a zero-mean, serially uncorrelated real aggregate supply shock. In order to avoid some well known difficulties which arise when the problem of central bank transparency is introduced into the model, and to obtain closed form solutions, throughout the paper we normalize $\alpha = 1$, in line with the approach followed by most of the recent literature on central bank transparency (see, e.g., Geeratz [20]; Hughes-Hallet and Viegi [24]; Kobayashi [26]; Demertzis, Hughes Hallett and Viegi [13]; Muscatelli [29] and [30]; Demertzis and Hughes Hallett [12]).

As for the aggregate demand side, we employ the formulation adopted by Walsh [35]:

$$\pi = m + v - \gamma\varepsilon \tag{2}$$

where $m$ is the rate of growth of the money supply (controlled by the central bank), $v$ is either a control error or a velocity shock (with $E(\nu) = 0, E(\nu^2) = \sigma_v^2$ , and $E(v\epsilon) = 0$), and the term $\gamma\varepsilon$ represents the (negative, as $\gamma > 0$) impact of the real supply shock on inflation. Equation (2) allows us to explicitly consider the influence of the contracts on the bank's choice of its instrument.

In a standard Barro-Gordon model - without the imperfect transparency problem and the incentive contract - the central bank loss function would be: $L^{CB} = \left(y - y^{CB}\right)^2 + \beta\pi^2$, so that society's aversion to inflation and the central banker's degree of conservatism (inflation aversion) would be both equal to $\beta$, while the output target of the monetary authorities would be equal to $y^{CB} = y^n + z$ (the term $z \geq 0$ represents the expansionary bias of the monetary authorities). In the present analysis, the private sector, represented by the government, can influence the central banker's behavior through a linear incentive scheme that penalizes deviations of actual inflation from its target. Following Walsh [35], the incentive scheme is composed of a fixed reward $t_0$ and a marginal penalty $t$ applied to the realized inflation rate, so that the form of the scheme is:[9] $t_0 - t\pi$. The presence of a linear incentive scheme modifies the central bank's loss function $L^{CB}$ along the lines of Walsh [35].

Following CM [7], a second principal can be included into the analysis; this third player represents a disaffected interest group endowed with own objectives which are in conflict with those of the "general" public (society). As mentioned in the introduction, the objectives of the interest group can differ from those of society in a number of dimension. Here we will consider only what appears to be the most realistic (and probably empirically relevant) case, i.e., that of an interest group having a greater output target than that of society, set equal to the natural level $y^n$. The output target of the interest group is taken to be equal to: $y^g = y^n + g$, with $g > 0$. This assumption seems a natural one since, as discussed in CM [7], a number of relevant aggregate players interested in higher output can be easily imagined.[10] The interest group offers a contract based on the observed output values: it assigns to the central banker a given payment $\tau_0$ minus a penalty equal to $\tau\left(y^g - y\right)$. The interest group's contract is thus: $\tau_0 - \tau\left(y^g - y\right)$.

[9] Walsh [35] showed that an optimal penalty rate exists that produces the same policy outcomes as those that would arise under credible commitment. This penalty rate rises the marginal costs for the central banker of higher money growth rates, thus inducing her to eliminate the inflation bias.

[10] CM [7] point out a problem with this interest group which is present also in our analysis. Since in our model - as in that of CM [7] - expectations are always satisfied in the final equilibrium, the consequent output level is always equal to the natural one, and the central banker is always punished. Her effort to increase output beyond the natural value is hence futile. We will discuss this issue more in detail in the following sections.

The timing of the game is as follows. At the beginning of the period, the government offers a contract that makes the payment to the central banker contingent on inflation performance, the interest group offers a competing incentive scheme that makes the central banker's reward contingent on output performance and the "nature" chooses the central bank's preference parameter, which is unknown to the two principals. Then the private sector forms rational price expectations (not knowing the value of the central bank's preference parameter, which is opaque to the public), the stochastic supply shock takes place and the central banker sets the policy instrument. Finally, the stochastic control shock $v$ occurs, inflation and output outcomes are realized and the central banker is rewarded. It should be noted that, in line with CM [7] but differently from Walsh [35], monetary policy does not react to the shock $v$, which was there taken as observed by the central bank, whereas it is here assumed to be stochastic for all players.

The central bank loss function, $L^{CB}$, combines the objective function chosen by CM [7], and the standard function with imperfect transparency, based on the assumption that monetary policy is delegated to a central banker who is randomly selected from society (Beetsma and Jensen [2]; [3]):[11]

$$L^{CB} = \left[(1+\lambda)(y-y^n-z)^2 + (\beta-\lambda)\pi^2\right] - \xi\left[\tau_0 - \tau(y^n+g-y) + t_0 - t\pi\right] \quad (3)$$

where $\xi$ is a parameter representing the trade-off between the contract benefits and the loss for the central banker due to inflation and output deviations from the respective targets.

Uncertainty on the central banker's preferences is represented by the random variable $\lambda \in [-1, \beta]$, where $E(\lambda) = 0$, $E(\lambda) = \sigma_\lambda^2$, and $E$ is the expectation operator. In this section we focus our attention to the so-called "contingent", or "political", view of transparency (Eijffinger and Geraats [16]; Posen [32]; Hughes-Hallet and Viegi [24]) - i.e., that related to information asymmetries between central bank and the general public about the weight of the arguments in the monetary authorities' objective functions - which relates transparency to the CB's degree of conservativeness.[12] According to equation (3), the level of uncertainty associated with CB's preferences is represented by the variance $\sigma_\lambda^2$. As the random variable $\lambda$ takes values in a compact set and has expected value equal to zero, $\sigma_\lambda^2$ must have a well defined upper bound; more precisely:[13]: $\sigma_\lambda^2 \in [0, \beta]$. Finally, note that $\lambda$, $\varepsilon$ and $v$ are uncorrelated.

In line with the procedure adopted by Chortareas and Miller [7], the central bank solves the problem:

$$\begin{aligned} \min_m EL^{CB} &= E\left[(1+\lambda)(y-y^n-z)^2 + (\beta-\lambda)\pi^2\right] - E\xi\left[\tau_0 - \tau(y^n+g-y) + t_0 - t\pi\right] \\ \text{s.t. } \pi &= m + v - \gamma\varepsilon; \qquad y = y^n + (\pi - \pi^e) + \varepsilon \end{aligned}$$

[11] We choose this specification because it avoids the arbitrary effects of CB's preference uncertainty on average monetary policy documented by Beetsma and Jensen [3].

[12] See, among others, Cukierman [11], Geraats [20], Gersbach and Hahn [21] [22].

[13] For a simple demonstration of this result, see Ciccarone et. al. [10], Appendix.

Recalling that the velocity shock $\nu$ is stochastic for both the public and the central banker, and that she can observe the exact value of the supply shock $\varepsilon$, the reaction function is:

$$m = \left(\gamma - \frac{1+\lambda}{1+\beta}\right)\varepsilon + \left(\frac{1+\lambda}{1+\beta}\right)\pi^e + \left(\frac{1+\lambda}{1+\beta}\right)z + \frac{\xi(\tau - t)}{2(1+\beta)} \tag{4}$$

The public's inflation expectations are: $\pi^e = E[\pi] = E[m + v + \gamma\varepsilon]$; using equation (4) and recalling that $\varepsilon$ and $\lambda$ are uncorrelated, we obtain:

$$\pi^e = \left(\frac{1}{\beta}\right)z + \left(\frac{\xi}{2\beta}\right)(\tau - t) \tag{5}$$

According to the timing of the game moves, the interest group sets its choice variable $\tau$ knowing the reaction function of the central bank (4) and the value of the private sector expectations (5); its optimization problem is then:

$$\begin{aligned} \min_{\tau} E\left(L^{IG}\right) &= E\left[(y - y^g)^2 + b\pi^2\right] + \psi E\left[\tau_0 - \tau(y^g - y)\right] \\ &\quad \text{s.t. equations (1); (2); (4); (5)} \end{aligned}$$

Parameter $\psi$ represents the interest group's relative concern about the contract costs and the loss due to deviation form its $y$ and $\pi$ targets, while $b$ is the inflation aversion of the same interest group. By substituting $m$ and $\pi^e$ into the equations of aggregate demand and supply (so as to obtain values for $y$ and $\pi$), the optimization problem can be rewritten as:

$$\begin{aligned} \min_{\tau} EL^{IG} &= E\left[\left(1 - \frac{(1+\lambda)}{1+\beta}\right)\varepsilon + \frac{\lambda}{\beta}z + v + \frac{\lambda\xi}{2\beta(1+\beta)}(\tau - t) - g\right]^2 + \\ &\quad + Eb\left[\left(-\frac{1+\lambda}{1+\beta}\right)\varepsilon + \left(\frac{1+\lambda}{\beta}\right)z + v + \left(\frac{(1+\lambda+\beta)\xi}{2\beta(1+\beta)}\right)(\tau - t)\right]^2 + \\ &\quad + E\psi\left[\tau_0 - \tau(y^g - y)\right] \end{aligned}$$

The optimality condition for the interest group turns out to depend on the variance $\sigma_\lambda^2$, i.e. on $E\left(\lambda^2\right)$, and the reaction function is:

$$\tau = \left(\frac{2(1+\beta)}{\xi^2}\right)\frac{(1+\beta)\left[\beta^2\psi g - z\xi b\right] - z\xi(1+b)\sigma_\lambda^2}{b(1+\beta)^2 + (1+b)\sigma_\lambda^2} + t \tag{6}$$

It is straightforward to verify that by setting $\sigma_\lambda = 0$ the same reaction function as CM [7] is obtained: $\tau|_{\sigma_\lambda=0} = \frac{2}{\xi}\left(\frac{\beta^2\psi g}{\xi b} - z\right) + t$.

A first observation concerning equation (6) has to do with the reaction of $\tau$ (given the incentive $t$ supplied by the government) to an increase in the degree of opacity $\sigma_\lambda^2$. It is easy to check that this reaction is negative: $\frac{\partial\tau}{\partial\sigma_\lambda^2} < 0$ An increase in preference uncertainty makes it less convenient for the interest group to strongly incentivize the central banker with a high penalty rate. This is not surprising: coherently with Brainard's [4] principle, higher uncertainty (the increase in "political" opacity) induces moderation in the use of the instrument.

The behavior of the government is described by the solution of the problem:

$$\begin{aligned}\min_t E\left(L^G\right) &= E\left[(y-y^n-z)^2+\beta\pi^2\right]+\phi E\left[t_0-t\pi\right] \\ &\text{s.t. equations (1); (2); (4); (5)}\end{aligned}$$

where $\phi$ is the weight attached by the government to the cost of providing the incentive, relative to the loss deriving from the deviations of $y$ and $\pi$ from their respective targets.[14] By proceeding as in the interest group's case, we can compute the government's reaction function:

$$t=\left(\frac{2z(1+\beta)}{\xi}\right)\left[\frac{\beta(\phi+\xi)+\xi\sigma_\lambda^2}{\xi\sigma_\lambda^2+(2\phi+\xi)\beta(1+\beta)}\right]+\frac{\beta(1+\beta)(\phi+\xi)+\xi\sigma_\lambda^2}{\xi\sigma_\lambda^2+(2\phi+\xi)\beta(1+\beta)}\tau \tag{7}$$

It is again straightforward to verify that by setting $\sigma_\lambda=0$, it reduces to that found by CM [7]: $t|_{\sigma_\lambda=0}=\frac{\phi+\xi}{2\phi+\xi}\left[\frac{2z}{\xi}+\tau\right]$. Differently from the interest group's reaction, the response of the government to an increase in opacity $\sigma_\lambda$ (for given $\tau$) seems to depend in complicate ways on parameter values. We shall however show in the next subsection that if the government's concern for inflation is greater than that of the interest group, greater opacity leads to a decrease in the equilibrium penalty rates of both principals.

## 1.2 Equilibrium outcomes and the dominance of the interest group's contract

Using the reaction functions (6) and (7) we calculate the Nash solution for the interest group's incentive:

$$\tau^N=\left(\frac{2\beta}{\xi^2\beta\phi}\right)\left[\frac{(1+\beta)\left[\beta\psi g+\xi zb\right]\left[\beta(1+\beta)(2\phi+\xi)+\xi\sigma_\lambda^2\right]}{b(1+\beta)^2+(1+b)\sigma_\lambda^2}-\left[\phi+(2\phi+\xi)\beta\right]z\xi\right]$$

If $\sigma_\lambda=0$, we get again CM's result: $\tau^N=\left(\frac{2}{\xi\phi}\right)\left[\frac{(2\phi+\xi)\beta^2\psi g}{\xi b}-\phi z\right]$. Thus, in the case of full transparency, the condition $\tau^N>0$ requires to satisfy the inequality: $\beta^2(2\phi+\xi)\psi g-b\xi\phi z>0$. When it is instead $\sigma_\lambda^2>0$, in order to have a positive value of $\tau^N$ it must be:

$$\left[(1+\beta)\beta\psi g+(b-\beta)\xi z-(1+2\beta)(1+b)\phi z\right]\xi\sigma_\lambda^2+(1+\beta)^2\left[\beta^2(2\phi+\xi)\psi g-b\xi\phi z\right]>0$$

[14] Note that, as in Walsh [35], the central bank and the government share the same output and inflation targets.

Although the positivity of $\tau^N$ depends in a complicated way on parameter values (relative to $\sigma_\lambda^2$), comparability with CM requires to take that the inequality $\beta^2 (2\phi + \xi) \psi g - b\xi\phi z > 0$ holds. The condition hence becomes:

$$\frac{(1+\beta)\beta\psi g + (b-\beta)\xi z - (1+2\beta)(1+b)\phi z}{(1+\beta)^2 \left[\beta^2 (2\phi+\xi)\psi g - b\xi\phi z\right]}\xi > -\frac{1}{\sigma_\lambda^2}$$

This inequality implies that, under imperfect political transparency, $\tau^N > 0$ requires that $\psi$ and $g$ are large enough in comparison to $\phi$ and $z$: the interest group must have a greater concern for output (and a greater concern for the incentive cost) than the government.

By using the expression for $\tau^N$, we can calculate the equilibrium value for the government incentive and summarize the Nash equilibrium between the government and the interest group as:

$$\tau^N = \left(\frac{2}{\xi^2\phi}\right)\left[(1+\beta)\left[\beta\psi g + \xi z b\right]\frac{\Omega}{\Lambda} - \left[\phi + (2\phi+\xi)\beta\right] z\xi\right] \quad (8)$$

$$t^N = \left(\frac{2}{\xi^2\phi\Lambda}\right)\left\{\beta(1+\beta)\left[\Omega - \phi\beta(1+\beta)\right]\psi g + \left[(b-\beta)\xi - \beta(1+b)\phi\right]\xi\sigma_\lambda^2 z\right\} \quad (9)$$

where:

$$\Omega = \left[\beta(1+\beta)(2\phi+\xi) + \xi\sigma_\lambda^2\right]; \qquad \Lambda = b(1+\beta)^2 + (1+b)\sigma_\lambda^2$$

The reactions of $\Omega$ and $\Lambda$ to opacity are: $\frac{\partial\Omega}{\partial\sigma_\lambda^2} = \xi > 0$; $\frac{\partial\Lambda}{\partial\sigma_\lambda^2} = 1 + b > 0$; these allow us to analyze the effects of opacity on the equilibrium incentive rates:

$$\frac{\partial\tau^N}{\partial\sigma_\lambda^2} = \left(\frac{2(1+\beta)^2(\beta\psi g + \xi z b)}{\xi^2\phi\Lambda^2}\right)\left[\xi(b-\beta) - 2\beta(1+b)\phi\right]$$

$$\frac{\partial t^N}{\partial\sigma_\lambda^2} = \left(\frac{2(1+\beta)^2(\beta\psi g + b\xi z)}{\xi^2\phi\Lambda^2}\right)\left[\xi(b-\beta) - \beta(1+b)\phi\right]$$

The reactions of $\tau^N$and $t^N$ to opacity are hence ruled by the relative values of the inflation aversion parameters of the government (and central bank, on average) and of the interest group:

$$\frac{\partial\tau^N}{\partial\sigma_\lambda^2} \gtreqless 0 \Leftrightarrow \frac{1}{\beta} + 1 \gtreqless \left(\frac{1}{b}+1\right)\left(\frac{2\phi}{\xi}+1\right)$$

$$\frac{\partial t^N}{\partial\sigma_\lambda^2} \gtreqless 0 \Leftrightarrow \frac{1}{\beta} \gtreqless \frac{1}{b} + \left(\frac{1}{b}+1\right)\frac{\phi}{\xi}$$

If the inflation aversion of the government, $\beta$, is high enough, the incentives of both the government and the interest group will tend to decrease when opacity increases.

In general, it appears reasonable to assume $\beta \geq b$, i.e., the government/society has a greater (or equal) concern for inflation than the expansionary lobbyist;[15] in this case the impact of $\sigma_\lambda$ is unequivocal: greater opacity leads to a decrease in the penalty rates of both principals ($\partial\tau^N/\partial\sigma_\lambda^2$ and $\partial t^N/\partial\sigma_\lambda^2$ are both negative). When the two principals experiment an increase in the common source of uncertainty related to the "type" of central banker to whom monetary policy is delegated, Brainard's moderation principle leads them to reduce their penalty rates. The government and the interest group act more carefully when they know less about the way in which the central bank might react to their incentives.[16]

Note however that the final equilibrium outcomes for the macroeconomic (observable) variables do not depend on the values of $\tau^N$ and $t^N$ *per se*, but rather on their difference $\left(\tau^N - t^N\right)$, as shown by the following equations:

$$\begin{aligned}
\pi^e &= \left(\frac{1}{\beta}\right) z + \left(\frac{\xi}{2\beta}\right)\left(\tau^N - t^N\right) \\
\pi &= \left(-\frac{1+\lambda}{1+\beta}\right)\varepsilon + \left(\frac{1+\lambda}{\beta}\right) z + \left(\frac{(1+\lambda+\beta)\,\xi}{2\beta\,(1+\beta)}\right)\left(\tau^N - t^N\right) + v \qquad (10) \\
y &= y^n + \left(1 - \frac{(1+\lambda)}{1+\beta}\right)\varepsilon + \frac{\lambda}{\beta} z + \frac{\lambda\xi}{2\beta\,(1+\beta)}\left(\tau^N - t^N\right) + v
\end{aligned}$$

These results are in line with the general findings of multiple agency theory. Since the two principals have conflicting aims and act non cooperatively, their incentives are substitute, not complement, that is, they influence outcomes in opposite ways.[17]

From equations (8) and (9) we obtain the value $\Delta_{rates}\left(\sigma_\lambda^2\right) = \tau^N - t^N$ as a function of opacity (and of the other parameters):

$$\tau^N - t^N = \Delta_{rates}\left(\sigma_\lambda^2\right) = \left(\frac{2\,(1+\beta)}{\xi^2\Lambda}\right)\left[(1+\beta)\left(\beta^2\psi g - b\xi z\right) - (1+b)\,\xi z\sigma_\lambda^2\right] \qquad (11)$$

The derivative of this expression with respect to opacity is:

$$\frac{\partial\Delta_{rates}\left(\sigma_\lambda^2\right)}{\partial\sigma_\lambda^2} = -\left(\frac{2\beta\,(1+\beta)^2\,(1+b)\,(b\xi z + \beta\psi g)}{\xi^2\Lambda^2}\right) < 0$$

[15] It could of course be possible to consider the opposite case of a conservative interest group highly concerned for inflation (i.e., the financial community). However we reckon this as less interesting case in a context where most of the central banks of the industrialised countries set values for their inflation targets around 2%.

[16] A possible interpretation of the stronger impact of $\sigma_\lambda^2$ on $\tau^N$ than on $t^N$ is related to the fact that the CB and the Government share the same concern for inflation ($\beta$) whereas that of the IG ($b$) is lower. It is well known since Brainard's seminal article that the (negative) reaction to an increase in $\sigma_\lambda^2$ is stronger the higher is weight attached to inflation in the player objective function.

[17] This behavior creates a negative externality, the value of which can be measures by solving for the cooperative equilibrium and by comparing the values of the resulting outcomes with those obtained in the Nash equilibrium.

This derivative shows that $\Delta_{rates}\left(\sigma_\lambda^2\right)$ is a decreasing function of $\sigma_\lambda^2$: as opacity increases, $\tau^N$ and $t^N$ get closer (further apart) when $\tau^N > t^N$ $(\tau^N < t^N)$.

By plugging expression (11) into equations (10) we get:

$$\begin{aligned}
(\pi^e)^* &= \left(\frac{1}{\beta}\right) z + \left(\frac{\xi}{2\beta}\right) \Delta_{rates}\left(\sigma_\lambda^2\right) \\
\pi^* &= \left(-\frac{1+\lambda}{1+\beta}\right) \varepsilon + \left(\frac{1+\lambda}{\beta}\right) z + v + \left(\frac{(1+\lambda+\beta)\,\xi}{2\beta\,(1+\beta)}\right) \Delta_{rates}\left(\sigma_\lambda^2\right) \\
y^* &= y^n + \left(1 - \frac{(1+\lambda)}{1+\beta}\right) \varepsilon + \frac{\lambda}{\beta} z + v + \frac{\lambda\xi}{2\beta\,(1+\beta)} \Delta_{rates}\left(\sigma_\lambda^2\right)
\end{aligned} \tag{12}$$

An increase in transparency (a decrease in opacity $\sigma_\lambda^2$) has an expansionary effect: it raises output $y^*$ as well as inflation (effective and expected). As we now turn to show, it is reasonable to assume that $\tau^N > t^N$; this allows to explain the expansionary effect of higher transparency as produced by an increase in the difference between the penalty rates set by the interest group and by the government. This result shows that the finding by Muscatelli [29] and by Beetsma and Jensen [2] that under single agency a stochastic inflation bias persists in equilibrium holds also under common agency

One crucial issue in our analysis is how the introduction of transparency affects the relative strength of the competing contracts offered by the two principals. In order to tackle this issue we check, along the lines of CM, if the interest group contract always dominates that of the government. To this aim, we set $t = 0$ in (6) so as to obtain:

$$\tau_{t=0}^N = \left(\frac{2\,(1+\beta)}{\xi^2\Lambda}\right) \left[(1+\beta)\left[\beta^2\psi g - \xi b z\right] - z\xi\,(1+b)\,\sigma_\lambda^2\right]$$

Thus the value of $\tau_{t=0}^N$ is identical to that of the difference $\Delta_{rates}\left(\sigma_\lambda^2\right)$ in equation (11), that is the difference between the penalty rates under multiple agency. This implies that the equilibrium values for output and inflation (expected/effective) when the government is absent, $y_{t=0}$, $\pi_{t=0}^e$ and $\pi_{t=0}$, are identical to those of equations (12):

$$(\pi^e)^* - \pi_{t=0}^e = \pi^* - \pi_{t=0} = y^* - y_{t=0} = 0$$

The presence of uncertain "political" transparency (as far as inflation and output are concerned) does not change CM's result that the interest group contract dominates the Government's one: the results obtained when both agents set their rewards are the same as those cropping up when only the interest group offers the incentive reward. This is indeed reasonable, as the kind of uncertainty faced by the two players is the same (none of them possesses an information advantage) and they set $\tau$ and $t$ under a Nash interaction. In this case the interest group also sets a $\tau$ which always dominates the government incentive $t$ in affecting the central bank's behavior.

Finally, expression (11) allows us to examine the relationship between opacity and the sign of the difference between the two penalty rates; $\tau^N$ is greater than $t^N$ if $\sigma_\lambda^2$ is small enough:

$$\tau^N - t^N \gtreqless 0 \quad \text{iff:} \quad \sigma_\lambda^2 \lesseqgtr \frac{(1+\beta)\left(\beta^2\psi g - b\xi z\right)}{(1+b)\,\xi z} \tag{13}$$

This condition is coherent with the idea that imperfect political transparency "softens" the problem of the inflationary bias induced by the dominance of the IG contract. Assuming that the condition guaranteeing in CM[7] that $\tau^N > t^N$ holds, i.e., $\beta^2\psi g > b\xi z$, it becomes possible, yet not guaranteed, to obtain the same result under imperfect political transparency. If it is indeed $\tau^N > t^N$ (this happens when opacity is relatively low), condition (13) also shows that a rise in $\sigma_\lambda^2$ closes the gap between the two penalty rates and reduces $(\pi^e)^*$and $\pi^*$ - as it can be straightforwardly checked from the first two equations (12).

### 1.3 Average inflation under uncertain "political" transparency

The result obtained in the previous section mirrors that of CM, but the situations with and without central bank preference uncertainty are not at all the same. To see the differences, it is first necessary to recall that the value of inflation expectations when $\sigma_\lambda^2 = 0$ (that computed in CM) is $(\pi^e)^{\det} = \beta\psi g/\xi b$ and compare it with the equilibrium value $(\pi^e)^*$ under preference uncertainty (from equations (12)). The difference between the two rates is:

$$(\pi^e)^{\det} - (\pi^e)^* = \frac{(1+b)\,\sigma_\lambda^2}{\Lambda}\left(\frac{\beta}{\xi b}\psi g + z\right) > 0$$

unless, of course, $\sigma_\lambda^2 = 0$.

Under preference uncertainty equilibrium expected inflation is lower than under perfect transparency: for the reason we highlighted above, when opacity increases above zero the difference $(\tau - t)$ decreases and the inflation bias falls. This result shows the role that imperfect transparency can play in a multiple agency environment for the conduct of monetary policy. The dominance of the interest group contract might suggest that opacity plays no relevant role; yet, under perfect transparency the interest group offers a reward $\tau$ that, when combined with the government's $t$, produces expected inflation (and average inflation) higher than under imperfect transparency. Central bank's opacity hence softens the problem raised by the dominance of the interest group contract, as average inflation tends to get closer to the value desired by society. Actual equilibrium inflation under opacity, $\pi^*$, may instead be greater or smaller than that under full transparency according to the realized value of the shock $\lambda$, which influences $\pi^*$ as shown by the second of equations (12). As $\pi^e$ measures also the average value of inflation, the presence of imperfect transparency can reduce the average inflation bias.

The introduction of imperfect "political" transparency brings in another relevant outcome, as the presence of a new parameter allows us to determine a value of opacity $\sigma_\lambda^2|_{\pi^e=0}$ able to bring $(\pi^e)^*$ to zero. Before proceeding in this direction, it should be noted that, being $\sigma_\lambda^2$ the variance of a random variable and not an instrumental variable in the hands of the central bank, the value $\sigma_\lambda^2|_{\pi^e=0}$, which is a punctual value within the interval

$[0, \beta]$, corresponds to a specific and very particular circumstance which, given the other parameter values, is very unlikely to materialize.

To determine this value, we calculate the commitment solution by setting $\xi = \lambda = 0$ in the central bank's reaction function (4), so as to obtain: $m = \left(\gamma - \frac{1}{1+\beta}\right)\varepsilon + \left(\frac{1}{1+\beta}\right)\pi^e + \left(\frac{1}{1+\beta}\right)z$. Setting $\pi^e = 0$ and $\beta \to \infty$ we then obtain:

$$m = \gamma\varepsilon; \quad \pi = v; \;\; y = y^n + v + \varepsilon$$

In the multiple agency-imperfect transparency setting, in order to have $(\pi^e)^* = 0$ the first of expressions (12) must be equated to zero. The value of $\sigma_\lambda^2$ which produces this result is equal to:

$$\left.\sigma_\lambda^2\right|_{\pi^e=0} = \frac{\beta(1+\beta)^2\psi g}{(1+b)\xi z} \tag{14}$$

When opacity reaches the value $\left.\sigma_\lambda^2\right|_{\pi^e=0}$, expected inflation exactly matches the value under credible commitment: $(\pi^e)^* = \pi^e_{\text{commit}} = 0$. This is a relevant departure from CM's common agency results, as in their model with full transparency it is $(\pi^e)^{\det} = \beta\psi g/\xi b > 0$, and it is hence never possible to eliminate a positive inflation bias.

However, as this value of opacity (14) which brings inflation expectations to zero must be economically meaningful, $\left.\sigma_\lambda^2\right|_{\pi^e=0}$ must lie in the admissible interval $[0, \beta]$ or, better, it must not exceed the upper admissible bound $\beta$. This restriction imposes bounds on the remaining model parameters. In our setting $\sigma_\lambda^2$ is also subject to another restriction, as it must allow for a meaningful (i.e. positive) equilibrium value of $\tau^N$. When opacity equals $\left.\sigma_\lambda^2\right|_{\pi^e=0}$, the value of $\tau^N$ given by equation (8) must be greater than zero: $\tau^N\left(\left.\sigma_\lambda^2\right|_{\pi^e=0}\right) > 0$ implies an additional restriction on the model parameters. By imposing the two conditions, we obtain in fact:

$$\begin{aligned}
\left.\sigma_\lambda^2\right|_{\pi^e=0} &= \frac{\beta(1+\beta)^2\psi}{(1+b)\xi}\frac{g}{z} < \beta \;\text{ iff }\; \frac{g}{z} < \frac{(1+b)\xi}{(1+\beta)^2\psi} \\
\tau^N\left(\left.\sigma_\lambda^2\right|_{\pi^e=0}\right) &> 0 \;\text{ iff }\; \frac{g}{z} > \frac{(1+b)\phi}{\beta(1+\beta)\psi}
\end{aligned}$$

Combining the two inequalities we can specify an interval for the admissible values of the ratio $g/z$:

$$\begin{aligned}
\tau^N &> 0 \wedge \left.\sigma_\lambda^2\right|_{\pi^e=0} < \beta \text{ requires:} \\
\frac{(1+b)\xi}{(1+\beta)^2\psi} &> \frac{g}{z} > \frac{(1+b)\phi}{\beta(1+\beta)\psi} \;\text{ and: }\; \frac{(1+b)\xi}{(1+\beta)^2\psi} > \frac{(1+b)\phi}{\beta(1+\beta)\psi}
\end{aligned}$$

From which we obtain the final set of restrictions:

$$\begin{aligned}
\tau^N &> 0 \wedge \left.\sigma_\lambda^2\right|_{\pi^e=0} < \beta \text{ iff:} \\
\text{I)} \quad \frac{(1+b)\,\xi}{(1+\beta)^2\,\psi} &> \frac{g}{z} > \frac{(1+b)\,\phi}{\beta\,(1+\beta)\,\psi} \\
\text{II)} \quad \frac{\beta}{1+\beta} &> \frac{\phi}{\xi}
\end{aligned}$$

Condition II), which must necessarily hold for condition I) to be possible, imposes a strong (and neatly interpretable) restriction on two key parameters: in order to obtain inflation expectations equal to zero with acceptable values for $\tau$ and $\sigma_\lambda^2$, the ratio $\frac{\phi}{\xi}$ must be less than 1: the weight attached by the government to the cost of its incentive scheme must be smaller than the weight attached by the central bank to the two schemes. An optimal degree of opacity exists only if the central bank cares for the incentive payment more than the government. The intuition for this result runs as follows: for inflation expectations to be equal to zero, $\Delta_{rates}\left(\sigma_\lambda^2\right) = \tau^N - t^N$ must be big enough (in absolute value) and negative; this implies that $t^N$ must be big enough, which is more likely when the cost of the incentive contract is small for the government, or when the central bank attaches to this incentive a high value.

The government plays an essential role in this context, as only its presence allows for the possibility to obtain the degree of opacity that guarantees $\pi^e = 0$. To see this point, note that, by calculating whether $\tau^N$ is positive when the government does not use the incentive contract (i.e. in the case of $t = 0$), it follows that in order to obtain $\tau_{t=0}^N = \left(\frac{2(1+\beta)}{\xi^2\Lambda}\right)\left[(1+\beta)\left(\beta^2\psi g - \xi bz\right) - z\xi\,(1+b)\,\sigma_\lambda^2\right] > 0$, it must be $-\beta\psi g > \xi bz$, which can never be true. Thus, when it is $\sigma_\lambda^2 = \sigma_\lambda^2|_{\pi^e=0}$ it is certainly $\tau_{t=0}^N < 0$. This shows that it is impossible to conceive a game in which the interest group contract dominates and where there exists a value of opacity able to bring inflation expectations to zero, unless the governments also acts in the economy by providing an incentive scheme to the central bank.

It is difficult to analyze the dependence of $\sigma_\lambda^2|_{\pi^e=0}$ on the model's fundamentals (basically, the preference parameters) as conditions I) and II) involve several model parameters. A simple numerical example can however show some of the effects of the variations in the central bank's average degree of conservatism ($\beta$) on $\sigma_\lambda^2|_{\pi^e=0}$. From inequality I) we note that the existence of an economically feasible value of $\sigma_\lambda^2|_{\pi^e=0}$ is more unlikely when the central bank is on average conservative; this remark is confirmed by two numerical examples shown in figure 1 below, where the expected inflation rate $(\pi^e)^*$ is plotted against opacity under two different parameterisations:

With parameterisation A), where the central bank is (perceived as) "populist" ($\beta = 0.12$), the opacity value ensuring an average inflation rate equal to zero lies inside the interval defined by conditions I) and II) and is equal to $\sigma_\lambda^2|_{\pi^e=0} = 0.1146$; this result requires that the central bank cares about the incentives twice as much as the interest group: the ratio $\frac{\xi}{\psi}$ is equal to 2. In parametrisation B) the central bank is conservative on average ($\beta = 2$) but the ratio $\frac{b}{\beta} = 0.75$ is the same as in parameterisation A) (on average,

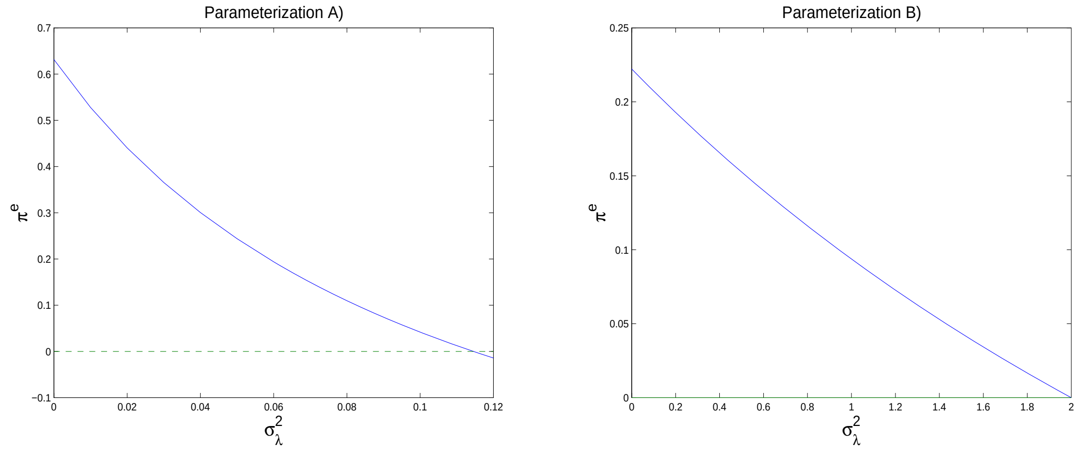


Figure 1: Expected inflation $(\pi^e)^*$ as a function of opacity $\sigma^2_\lambda$, for two different parameterisations: A) $\beta = 0.12, b = 0.095$, $\psi = 1, \xi = 2, g = 1, z = 0.6$; and B) $\beta = 2, b = 1.5$, $\psi = 1, \xi = 6, g = 1, z = 0.6$. Note that for both parameterizations the central bank and the government have an expansionary bias that is 40% lower than that of the interest group.

the interest group still cares about inflation less than the central bank). "Optimal" opacity $\sigma^2_\lambda|_{\pi^e=0}$ is now equal to the maximum possible value (i.e., $\beta = 2$), but in this extreme case the required $\frac{\xi}{\psi}$ is equal to 6 (three times larger than in the previous parameterisation). If $\frac{\xi}{\psi}$ where lower, $\sigma^2_\lambda|_{\pi^e=0}$ would lie outside the admissible interval defined by I)-II). A relatively high degree of conservatism may hence rule out the possibility of using opacity as a "discipline device" to counteract the expansionary pull of the interest group incentive scheme, as it may require an unrealistic value of $\frac{\xi}{\psi}$.

The numerical example of Figure 1 may help to explain some of the features of the observed behavior of central banks operating in different institutional set-ups with respect to political transparency. A result of the empirical research on transparency is that the European Central Bank (together with many other ones) has a index of political transparency significantly greater than that of, for example, the Central Bank of Japan[18]. Since the role and the presence of interest groups in Euro area and in Japan can be considered as equivalent (actually, it is probably stronger in the Euro area than in the other country) and the ECB is perceived, on average, as more conservative than the Bank of Japan, the above analysis suggests that, in order to achieve a low inflation rate, the ECB may be less opaque than the other central bank.

Finally, it should be stressed that the story we have told on $\sigma^2_\lambda|_{\pi^e=0}$ holds only for the average values of the endogenous macroeconomic variables. Writing $\pi^*$ as a function of $(\pi^e)^*$:

[18] See Eijffinger and Geraats [16]; for a principal components analysis on the Eijffinger and Geraats database (confirming the above mentioned results) see also Di Bartolomeo and Marchetti [14].

$$\pi^* = \left(-\frac{1+\lambda}{1+\beta}\right)\varepsilon + \left(\frac{\lambda}{1+\beta}\right)z + v + \frac{(1+\lambda+\beta)}{(1+\beta)}(\pi^e)^*$$

it is clear that when $\sigma_\lambda = \sigma_\lambda^2|_{\pi^e=0}$ the equilibrium values of (actual) inflation and output reduce to:

$$\begin{aligned}
\pi^* &= \left(-\frac{1+\lambda}{1+\beta}\right)\varepsilon + \left(\frac{\lambda}{1+\beta}\right)z + v \\
y^* &= y^n + \left(\frac{\beta-\lambda}{1+\beta}\right)\varepsilon + \left(\frac{\lambda}{1+\beta}\right)z + v
\end{aligned}$$

If $z > 0$, it can never be $\pi^* = v$, unless $\beta \to \infty$, or $\varepsilon = \lambda = 0$. The commitment solution can be obtained only on average, while in any period actual inflation is generally different from the commitment value.

# 2 Common agency and uncertain "economic" transparency

Following Muscatelli [30], case c), we now assume that the central bank's inflation aversion $\beta$ is perfectly known, whereas there exists uncertainty on its output target, i.e., what is known in the literature as "economic" transparency (see, e.g. Eijffinger and Geraats [16]). In this case the central bank output target is equal to: $y^{CB} = y^n + z + \theta$, where $\theta$ is a random variable, with $E(\theta) = 0$ and variance $\sigma_\theta^2$. The timing of the game is the same as in the previous model, and we continue to assume that $\alpha = 1$.

Since $\theta$ has zero mean and finite variance (and is uncorrelated with $\varepsilon$ and $v$), it appears coherent to assume that it takes values on a finite set: $\theta \in [\theta_{\text{low}}, \theta_{\text{high}}]$, with $\theta_{\text{low}} < 0$ and $\theta_{\text{high}} > 0$. A reasonable value for $\theta_{\text{low}}$ is $-z$: when this level of $\theta$ is realized, the central bank output target exactly matches the natural rate $y^n$. As for the determination of $\theta_{\text{high}}$, we can assume that in our model the interest group represents that fraction of society with the highest desired level of output (the maximum expansionary bias). $\theta_{\text{high}}$ can hence be set equal to $(g - z)$: when this value is realized, the central bank's output target $y^n + z + \theta_{\text{high}}$ will be given by the maximum expansionary bias: $y^n + g$. If $\theta \in [-z, (g - z)]$, the maximum value of the variance is[19] $\sigma_\theta^2 = \frac{1}{4}g^2$, so that $\sigma_\theta^2 \in [0, g^2/4]$.

The objective function of the central bank is:

$$L^{CB} = \left[(y - (y^n + z + \theta))^2 + \beta\pi^2\right] - \xi\left[\tau_0 - \tau(y^n + g - y) + t_0 - t\pi\right]$$

The rate of money growth is obtained by solving the following problem:

$$\begin{aligned}
\min_m EL^{CB} &= E\left[(y - y^n - z - \theta)^2 + \beta\pi^2\right] - E\xi\left[\tau_0 - \tau(y^n + g - y) + t_0 - t\pi\right] \\
&\quad \text{s.t. (1); (2)}
\end{aligned}$$

[19]This value is obtained by following the procedure suggested in Ciccarone et. al. [10].

From the first order condition we obtain the central bank reaction function:

$$m = \frac{\theta}{1+\beta} + \frac{1}{1+\beta}\left(\pi^e + z\right) - \left(\frac{1}{1+\beta} - \gamma\right)\varepsilon + \frac{\xi}{2\left(1+\beta\right)}\left(\tau - t\right) \tag{15}$$

Price expectations by private agents depends on $E\left(m\right)$: $\pi^e = E\left(\pi\right) = E\left(m + v + \gamma\varepsilon\right)$. By using equation (15) together with $E\left(\theta\right) = 0$ they turn out to be equal to:

$$\pi^e = \left(\frac{1}{\beta}\right) z + \left(\frac{\xi}{2\beta}\right) E\left(\tau - t\right)$$

This expression for price expectation is the same as that we found under uncertain "political" transparency (see 10). We can then express the rate of money growth, the inflation rate and the output as a function of $\left(\tau - t\right)$:

$$\begin{aligned} m &= \frac{\theta}{1+\beta} - \left(\frac{1}{1+\beta} - \gamma\right)\varepsilon + \left(\frac{1}{\beta}\right) z + \frac{\xi}{2\beta} E\left(\tau - t\right) \\ \pi &= \frac{\theta}{1+\beta} - \left(\frac{1}{1+\beta}\right)\varepsilon + \left(\frac{1}{\beta}\right) z + \frac{\xi}{2\beta} E\left(\tau - t\right) + v & (16) \\ y &= y^n + \frac{\theta}{1+\beta} + \left(\frac{\beta}{1+\beta}\right)\varepsilon + v & (17) \end{aligned}$$

We now calculate the interest group reaction function by solving the problem:

$$\begin{aligned} \min_{\tau} E\left(L^{IG}\right) &= E\left[\left(y - y^g\right)^2 + b\pi^2\right] + \psi E\left[\tau_0 - \tau\left(y^g - y\right)\right] \\ & \text{s.t. (16); (17)} \end{aligned}$$

By proceeding as in the previous section, we obtain the interest group reaction function:

$$\tau = \left(\frac{2}{\xi}\right)\frac{\beta^2\psi g - b\xi z}{b\xi} + t$$

which is exactly the same as that obtained by CM [7] without preference uncertainty.

The government's problem is:

$$\begin{aligned} \min_{\tau} E\left(L^{G}\right) &= E\left[\left(y - \left(y^n + z\right)\right)^2 + \beta\pi^2\right] + \phi E\left[t_0 - t\pi\right] \\ & \text{s.t. (16); (17)} \end{aligned}$$

and its reaction function is:

$$t = \frac{2\left(\xi + \phi\right)}{\xi\left(\xi + 2\phi\right)} z + \left(\frac{\xi + \phi}{\xi + 2\phi}\right)\tau$$

which is also identical to the expression obtained by CM [7].

It follows that under imperfect economic transparency the difference between the (Nash) equilibrium values of the penalty rates, $(\tau^{\text{ec}} - t^{\text{ec}})$, and the values for the endogenous variables are:

$$\begin{aligned}
\tau^{\text{ec}} - t^{\text{ec}} &= \frac{2}{\xi}\left(\frac{\beta^2 \psi g}{\xi b} - z\right) \\
\pi^{e(\text{ec})} &= \frac{\beta\psi}{\xi b} g \\
\pi^{\text{ec}} &= \frac{\theta}{1+\beta} + \frac{\beta\psi}{\xi b} g + v - \left(\frac{1}{1+\beta}\right)\varepsilon \\
y^{\text{ec}} &= y^n + \frac{\theta}{1+\beta} + \left(\frac{\beta}{1+\beta}\right)\varepsilon + v
\end{aligned}$$

Whereas the expected values are the same as those of CM, the actual values of inflation and output, $\pi^{\text{ec}}$ and $y^{\text{ec}}$, differ only for the presence in our model of the additive term $\theta/(1+\beta)$. In the presence of uncertainty on the output target, the solution differs from the full transparency case only for the presence of a term depending on $\theta$ and having expected value equal to zero: the two solutions are the same on average. Opacity on $y^n$ hence matters only for the issue of stabilization of output and of inflation around their average values, but has no effect on the inflationary bias problem. This was not the case under opacity on $\beta$, as in that context the value of $\pi^e$ was dependent on the variance of $\lambda$. This result is in line with Muscatelli's [30] finding that the introduction of the random variable $\theta$ in the single agency context does not alter the results with respect to the case of full "economic" transparency.

The reason for this result lies in the nature of the uncertainty represented by $\theta$. Its consideration amounts only to the introduction of an additional source of *additive* uncertainty into the model, and its role is hence equivalent to that of the other exogenous shock, $\varepsilon$ and $v$: since the players' optimization problems are linear-quadratic, the certainty equivalent principle applies. The introduction of a shock on $\beta$ ($\lambda$ in the previous model) alters instead the other players' expected evaluation of the central bank's marginal rate of substitution between $\pi$ and $y$, leading to a situation of *multiplicative* uncertainty where Brainard's principle applies.

# 3 Common agency and "selfishness" uncertainty

In this section we extend to the multiple agency context CM's [6] analysis of uncertainty on the value of the weight attached by the central bank to the incentive scheme, under the assumption made throughout this paper that $\alpha = 1$. To this aim we retain also their assumption that there exist two types of central bankers: the "benevolent" one cares only about social welfare ($\xi = 0$), whereas the "selfish" one responds to the incentive scheme ($\xi > 0$).

Denoting with $\rho$ the fraction of society that believes that $\xi > 0$, with $(1-\rho)$ the fraction believing that $\xi = 0$ and with $m^e_{S,\xi>0}$ and $m^e_{S,\xi=0}$ the expected rates of money growth when $\xi > 0$ and when $\xi = 0$, CM's [6] main results under single agency are as follows. Since the benevolent central banker ($\xi = 0$) brings about the discretion inflation rate ($m^e_{S,\xi=0} = \pi = z/\beta$), whereas the selfish one ($\xi > 0$) brings about the optimal contract inflation rate, which is equal to the commitment solution ($m^e_{S,\xi>0} = \pi = 0$), when the value of $\xi$ is uncertain, the expected rate of money growth (where the subscript $S$ indicates single agency) can be calculated as: $E\left(m_S\right) = \rho\left(m^e_{S,\xi>0}\right) + (1-\rho)\left(m^e_{S,\xi=0}\right)$, so that $\pi^e_S = (1-\rho)\left(z/\beta\right)$. Denoting with $m_{SB}$ and $\pi_{SB}$, respectively, the actual money growth and inflation under single agency when the benevolent central banker accepts the contract and then breaches it, CM [6] demonstrate that: $m_{SB} = \left(\gamma - \frac{1}{1+\beta}\right)\varepsilon + \left(\frac{1-\rho+\beta}{1+\beta}\right)\frac{z}{\beta}$ and $\pi_{SB} = \left(\frac{1-\rho+\beta}{1+\beta}\right)\frac{z}{\beta} - \left(\frac{1}{1+\beta}\right)\varepsilon + v$. Since it is: $\pi_{SB} - \pi^e_S > 0$, the inflation surprise makes actual output greater than $y^n$ plus the $\varepsilon$ shock.

In a context of common agency, we consider the same type of uncertainty on $\xi$ and treat it as a random variable $\xi \in \{0, \xi_1\}$, where $\xi_1 > 0$,with $prob\,(0) = (1-\rho)$ and $prob\,(\xi_1) = \rho$. It follows that: $E\left(\xi\right) = \rho\xi_1$ and $\sigma^2_\xi = \rho\xi_1^2$. The central banker knows the realization of the random variable, whereas the principals know only its distribution. The general reaction function of the central banker is:

$$m = \left(\gamma - \frac{1}{1+\beta}\right)\varepsilon + \left(\frac{1}{1+\beta}\right)\pi^e + \left(\frac{1}{1+\beta}\right)z + \frac{\xi\left(\tau - t\right)}{2\left(1+\beta\right)}$$

If she is actually benevolent ($\xi = 0$) and breaches the contract, the actual money growth will be given by $m = \left(\gamma - \frac{1}{1+\beta}\right)\varepsilon + \left(\frac{1}{1+\beta}\right)\pi^e + \left(\frac{1}{1+\beta}\right)z$, but the principals do not know this because they cannot observe the realized value of $\xi$. They must form expectations, which are given by:

$$\begin{aligned}
\pi^e &= E\pi = E\left(m + v + \gamma\varepsilon\right) \\
E\left(\pi\right) &= E\left(m\right) = E\left[\left(\gamma - \frac{1}{1+\beta}\right)\varepsilon + \left(\frac{1}{1+\beta}\right)\pi^e + \left(\frac{1}{1+\beta}\right)z + \frac{\xi\left(\tau - t\right)}{2\left(1+\beta\right)}\right]
\end{aligned}$$

Recalling that $E\left(\xi\right) = \rho\xi_1$, it is hence:

$$\pi^e = \left(\frac{1}{\beta}\right)z + \rho\frac{\xi_1}{2\beta}\left(\tau - t\right) = Em \tag{18}$$

Substituting equation 18 together with the general reaction function of the central banker into $\pi = m + v - \gamma\varepsilon$ we obtain the expressions for the macroeconomic endogenous variables, which contain both $\xi$ and $\xi_1$:

$$\pi = -\left(\frac{1}{1+\beta}\right)\varepsilon + \frac{z}{\beta} + \frac{\rho\xi_1 + \beta\xi}{2\beta\left(1+\beta\right)}\left(\tau - t\right) + v \tag{19}$$

$$\pi - \pi^e = -\left(\frac{1}{1+\beta}\right)\varepsilon + \left[\frac{\xi - \rho\xi_1}{2\left(1+\beta\right)}\right]\left(\tau - t\right) + v \tag{20}$$

$$y = y^n + \left(\frac{\beta}{1+\beta}\right)\varepsilon + \left[\frac{\xi - \rho\xi_1}{2\left(1+\beta\right)}\right]\left(\tau - t\right) + v \tag{21}$$

Now we solve the control problems of the principals, start with the interest group:

$$\begin{aligned}&\min_{\tau} E\left[(y-y^g)^2+b\pi^2\right]+\psi E\left[\tau_0-\tau\left(y^g-y\right)\right]\\&\text{s.t. } (19);\ (21)\end{aligned}$$

From the first order condition we obtain the following equation:

$$\tau=\left(\frac{2\left(1+\beta\right)^2}{\beta^2}\right)\frac{\beta^2\psi g-zb\rho\xi_1}{\Gamma\sigma_\xi^2}+t \tag{22}$$

were: $\Gamma=(1+b)\,\beta^2+\rho b\,(1+\beta)^2$. An increase in $\xi_1$ raises $\sigma_\xi^2$ and this, once again and for given value of $t$, decreases $\tau$: greater uncertainty makes the interest group more cautious in its instrument setting.

The Government problem is:

$$\begin{aligned}\min_{\tau} E\left(L^G\right) &= E\left[(y-(y^n+z))^2+\beta\pi^2\right]+\phi E\left[t_0-t\pi\right]\\&\quad \text{s.t. } (19);\ (21)\end{aligned}$$

which implies:

$$t=\frac{2\left(1+\beta\right)z}{\beta\xi_1+(1+\beta)\left(\rho\xi_1+2\phi\right)}+\left[\frac{\beta\xi_1+(1+\beta)\left(\rho\xi_1+\phi\right)}{\beta\xi_1+(1+\beta)\left(\rho\xi_1+2\phi\right)}\right]\tau \tag{23}$$

By solving the system (22)-(23), the Nash equilibrium values of $\tau$ and $t$ could be computed. The macroeconomic outcomes (19)-(21) depend however only on the difference $\tau-t$, which (in equilibrium) is given by:

$$\tau-t=\left(\frac{2\left(1+\beta\right)^2}{\beta^2}\right)\frac{\beta^2\psi g-zb\rho\xi_1}{\Gamma\sigma_\xi^2}$$

By applying the argument of section 1.1 we find that the interest group contract dominates the government's one also under "selfishness" uncertainty.[20] It should however be noted that in this environment in order to have $\tau-t>0$ it must be $\beta^2\psi g>zb\rho\xi_1$. This condition is similar to the condition $\beta^2\psi g>zb\xi$ under which $\tau-t>0$ in CM [7] were, however, $\xi$ is the perfectly known value denoting the degree of selfishness of the central banker. In the case of uncertainty we are here considering, $\rho\xi_1$ does not of course correspond to that $\xi$.

The resulting level of equilibrium inflation surprise (20) depends upon the realization of the variable $\xi$, so that there are two possible levels of $(\pi-\pi^e)$: one arises when the central bank is actually benevolent ($\xi=0$, the case discussed in CM [6]) and the other one when the central bank is selfish ($\xi=\xi_1>0$).

[20] The equilibrium value of $\tau-t$ is equal to the equilibrium value of $\tau$ computed in the absence of the government, i.e., when $t=0$.

When $\xi = 0$ the equilibrium inflation surprise is:

$$\pi - \pi^e = -\left(\frac{1}{1+\beta}\right)\varepsilon - \rho\xi_1\left[\left(\frac{1+\beta}{\beta^2}\right)\frac{\beta^2\psi g - zb\rho\xi_1}{\Gamma\sigma_\xi^2}\right] + v$$

Recalling that $E(\xi) = \rho\xi_1$ and disregarding the random shocks $\varepsilon$ and $v$, $\pi - \pi^e$ may be negative. This occurs when $\beta^2\psi g > zb\rho\xi_1$, i.e., when $\tau - t > 0$. Contrary to CM's [6] finding (proposition 1), in a common agency set up under "selfishness" uncertainty, as long as a fraction of population believes that the central banker is selfish ($\rho > 0$), a benevolent central banker who masquerades as selfish may hence generate a negative inflation surprise (inflation expectations are higher than actual inflation) and an output lower than the natural rate $y^n$.

This result is due to the interplay of two distinct effects. First, under common agency the incentive provided by the interest group prevails, inducing a selfish central banker to expand inflation. Second, the public sector, when forming its expectations, must take into account the possibility that the central banker is selfish, in which case she would accept the contract $\tau$ and increase inflation. The combination of the two effects leads to a high level of $\pi^e$.

Finally, in order to determine whether it is convenient for a benevolent central banker to masquerade as selfish and then breach the contracts, we must calculate the value of her loss in this case and compare it with the loss she obtains when she does not masqueraded as selfish, refuses to subscribe the incentive contract and a truly selfish central banker determines the rate of money growth. Recalling that $y = y^n + \left[\frac{\xi - \rho\xi_1}{2(1+\beta)}\right](\tau - t) + v + \left(\frac{\beta}{1+\beta}\right)\varepsilon$, when $\xi = 0$ the expected loss will be equal to:

$$E\left(L_{\xi=0}^{bCB}\right) = \frac{(\rho\xi_1)^2}{4(1+\beta)}(\tau - t)^2 + \left(\frac{1+\beta}{\beta}\right)z^2 + \left[\frac{(1-\beta)z\rho\xi_1}{\beta(1+\beta)}\right](\tau - t) + 2\sigma_v^2 + \frac{(1+\beta^2)}{(1+\beta)^2}\sigma_\varepsilon^2$$

When $\xi = \xi_1$ the loss will instead be equal to:

$$E\left(L_{\xi=\xi_1}^{bCB}\right) = \frac{(\rho+\beta)^2\xi_1^2}{4\beta^2(1+\beta)}(\tau - t)^2 + \left(\frac{1+\beta}{\beta}\right)z^2 + \left[\frac{z\rho\xi_1}{\beta}\right](\tau - t) + 2\sigma_v^2 + \frac{(1+\beta^2)}{(1+\beta)^2}\sigma_\varepsilon^2$$

It follows that:

$$E\left(L_{\xi=\xi_1}^{bCB} - L_{\xi=0}^{bCB}\right) = \frac{\left[(\rho+2\beta)\rho + \beta^2(1-\rho)\right]\xi_1^2}{4(1+\beta)\beta^2}(\tau - t)^2 + (2z\rho\xi_1)(\tau - t)$$

If $\beta^2\psi g > zb\rho\xi_1$, then it is certainly $\tau - t > 0$, $\pi - \pi^e < 0$ and $E\left(L_{\xi=\xi_1}^{bCB} - L_{\xi=0}^{bCB}\right) > 0$. This is however only a sufficient condition: the latter inequality may hold, for some parameter values, also when $\tau - t < 0$ and $\pi - \pi^e > 0$. This qualifies CM's [6] findings: under single agency it is always convenient for a benevolent central banker to pretend to be selfish and subsequently breach the incentive contracts, and the inflation surprise is always positive, whereas under common agency it is not always convenient for the central banker to first accept and then breach the contract, and when she does so the inflation surprise may be either positive or negative.

# 4 Conclusions

In this paper we have studied the effects of linear contracts under common agency and three types of central bank preference uncertainty. We have highlighted that in this environments multiplicative uncertainty produces new results, whereas additive uncertainty brings about the same equilibrium outcomes already found in the literature.

Under imperfect "economic" transparency a random variable is simply added to the central bank's output target and the certainty equivalent principle continues to apply. Under multiplicative uncertainty, the degree of opacity influences players' behavior and induces them to be more cautious in their instrument setting (Brainard's principle applies). The interest group always offers a more "convincing" linear contract because its objectives are further away than the government's objectives from those of the central bank. When opacity increases, the incentives provided by both principals decrease and the average inflation bias falls, being always lower than under perfect transparency. Finally, there exists a value of opacity for which inflation expectations are equal to zero, but the commitment solution can be obtained only on average.

Multiplicative uncertainty under common agency plays a relevant role also when the central banker's degree of "selfishness" is not perfectly known. In this case, it is not always convenient for a benevolent central banker to pretend to be selfish, accept the contract and then breach it. Contrary to the conclusion reached under single agency, when she masquerades as selfish the inflation surprise may be negative and output may be lower than the natural rate. This result is due to the interplay of two effects: (i) the incentive provided by the interest group may prevail, inducing the selfish central banker to expand inflation; (ii) the public forms inflation expectations taking into account the possibility that the central banker is selfish, in which case she would accept the contract offered by the interest group, expanding inflation. The combination of the two effects leads to a high level of expected inflation.

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