Optimal trend inflation under inflation targeting*

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Abstract

General equilibrium models with nominal rigidities cannot find a rationale for the optimality of positive inflation targets. Our key contribution is that a long-term trade-off between inflation and output efficiency may exist in the long run due to real money balance effects.

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1 Introduction

Over the last decade central banks have shifted to a policy of announcing non-zero inflation targets. As acknowledged in Schmitt-Grohé and Uribe (2004) the justification for this policy is however hardly found in micro-founded models. Our paper faces this problem by investigating the long-run performance of DSGE New Keynesian model, i.e. its steady-state properties, under an inflation targeting regime. Although, in fact, New Keynesian models are providing new perspectives on stabilization policies,1 less attention has been placed on the their long-run properties as long-term markups are usually considered to be exogenous by assuming as given monopolistic distortions in the labor and goods markets. The assumption of exogenous markups has several negative shortcomings. For instance, little can be say about the long-term inflation, which is typically assumed to be constant and zero despite overwhelming evidence of

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1See Clarida et al. (1999), Woodford (2003), Christiano et al. (2005), Gali and Gertler (2007).
the contrary, and potential relationship between the short and long run output gaps both in levels and variances.

The figure below illustrates the inflation behavior for the United States. Trend inflation is clearly always different from zero in both simple and moving average.

![Figure 1 — Inflation in the United States (1966Q1-2005Q3).](image)

In micro-foundations context, Khan et al. (2003) and Schmitt-Grohé and Uribe (2004, 2007), Ropele and Ascarì (2007) are exceptions to the usual procedure of log-linearizing around a zero steady state. Khan et al. (2003) show that the optimal long-run inflation rate is actually negative, because a negative rate balances the benefits of following the Friedman rule and the costs of relative price distortions. Schmitt-Grohé and Uribe (2007) perform a similar exercise in a medium-scale model incorporating fiscal policy and many distortions. They find that optimal monetary policy can differ from the Friedman rule, but trend inflation is still negative, and only if one assumes that lump-sum taxes are available. Schmitt-Grohé and Uribe (2004) instead consider optimal monetary and fiscal policy by treating the level of trend inflation as exogenously calibrated to US post-war data. Ropele and Ascarì (2007) consider how optimal monetary policy is affected by changes in trend inflation. They extend the standard DSGE New Keynesian model to allow for positive trend inflation, which is however introduced exogenously. They find that even low trend inflation has strong effects on optimal monetary policy and the dynamics of inflation, output, and interest rates.

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3 For a discussion about this issue, see, among others, Goodfriend (1997); Martin and Rogers (1997), Woodford (2003), Blackburn and Pelloni (2004, 2005), and Benigno and Ricci (2008).

4 The blue line is the period mean and red line is the average moving mean (5 years).

5 The same is true in Neiss (1999). In our model, negative inflation is efficient, but as we will show is not optimal under inflation targeting.
rates. By considering discretionary policy, efficiency deteriorates and there is no guarantee of determinacy. Moreover, commitment, targeting non-zero trend inflation, also leads to substantial welfare losses. All of them exogenously assume trend inflation.\footnote{Apart from Schmitt-Grohé and Uribe (2007).}

By explicitly considering micro-foundations and a DSGE context, Benigno and Ricci (2008) show that, when wage setters take into account the future consequences of their current wage choices in the presence of downward nominal wage rigidities, a long-run Phillips curve relates average unemployment to average wage inflation; the curve is virtually vertical for high inflation rates but becomes flatter as inflation declines. Several results about volatilities are also derived. However, downward nominal wage rigidities are, once again, exogenously introduced.\footnote{They also find that macro-economic volatility shifts the Phillips curve outward, implying that stabilization policies can play an important role in shaping the trade-off. Moreover in their model nominal wages tend to be endogenously rigid also upward, at a low inflation. And when inflation decreases, the volatility of unemployment increases whereas the volatility of inflation decreases (this implies a long-run trade-off also between the volatility of unemployment and that of wage inflation).}

In this paper we consider a general equilibrium model with monopolistic competition in the goods and labor markets, developed by Neiss (1999), where a staggered timing structure in the acquisition of nominal money balances within a money-in-the-utility function framework generates a discretionary inflation equilibrium. We extend Neiss (1999) to inflation targeting. To simplify the analysis we assume full price flexibility in the goods market, whereas wages are predetermined. As in Gnocchi (2006a) and (2006b) we consider endogenous markups but we assume neither zero trend inflation nor non-atomistic wage setters. Differently from Schmitt-Grohé and Uribe (2004, 2007) and Ropele and Ascari (2007) we do not assume positive trend inflation, but derive it from the optimization of the central bank. In our context in fact inflation is optimal as in Khan et al. (2003), but positive, as we show that although negative inflation is efficient for the central bank it is optimal a commitment to a positive rate because the endogenous markup ultimately depends on the central bank’s target.

The rest of the paper is organized as follows. The next section outlines our baseline model. Section 3 derives and explains our main result, i.e. a positive inflation target disciplines wage setters. Section 4 evaluates welfare and derives the optimal target. A final section concludes.

\section{The baseline model}

The representative household \((i)\) maximizes the following utility function

\[ U = \sum_{t=0}^{\infty} \beta^t \left( \ln C_{t,i} - \eta \bar{r}_{t,i} + \frac{\gamma}{1 - \varepsilon} \left( \frac{M_{t,i}}{P_t} \right)^{1 - \varepsilon} \right) \]

(1)
where $\beta \in (0, 1)$ is the intertemporal discount rate, $C_{t,i}$ is a consumption bundle, $l_{t,i}$ is a differentiated labor type that is supplied to all firms, $\frac{M_{t,i}}{P_t}$ denotes real money holdings. Consumption basket and price index are defined as follows:

$$C_t = \left( \int_0^1 C_t(j) \, dj \right)^\frac{1}{\beta}$$

and

$$P_t = \left( \int_0^1 P_t(i) \, \frac{\sigma}{\sigma - 1} \, di \right)^{\frac{\sigma}{\sigma - 1}}.$$

The flow budget constraint is:

$$C_{t,i} + \frac{M_{t+1,i}}{P_{t+1}} + \frac{B_{t+1,i}}{P_t} = \frac{w_{t,i}}{P_t} \cdot l_{t,i} + \frac{M_{t,i}}{P_t} + \frac{\xi_t}{P_t} + \theta_t + R_t \cdot \frac{B_{t,i}}{P_t}$$

where $B_{t,i}$ denotes holdings of one-period bonds; $w_{t,i}$ is the nominal wage; $\xi_t$ is a lump-sum transfer from central bank profits, $\theta_t$ denotes firms profits, $R_t$ is the nominal interest rate. Note that $M_{t+1,i}$ is chosen at $t$.

The representative household $(i)$ chooses consumption and nominal money holding. By contrast, we assume that wage are set by atomistic labor unions or a single non-strategic union (as in Schmitt-Grohé and Uribe, 2004), which supply labor monopolistically to a continuum of labor markets of measure 1 indexed by $j \in [0, 1]$. In each labor market $j$, the union faces a demand for labor given by $\left( \frac{w_j^t}{w_t} \right)^{-\sigma} l_j^t$, where $w_j^t$ denotes the nominal wage charged by the union in labor market $j$ at time $t$, $w_t$ is an index of nominal wages prevailing in the economy, and $l_j^t$ is a measure of aggregate labor demand by firms.

In the goods market, there is a continuum of monopolistically competitive firms uniformly distributed over the interval $[0, 1]$. Each firm $(j)$ produces a differentiated good using a Cobb-Douglas production function:

$$y_t(j) = l_t(j)^{\alpha}$$

where

$$l_{t,j} = \left[ \int_0^1 l_{t,j}(i) \, \frac{\sigma}{\sigma - 1} \, di \right]^{\frac{\sigma}{\sigma - 1}}$$

denotes a labour bundle and $\sigma$ is the intra-temporal elasticity of substitution across different labor inputs.

The central bank directly controls the nominal interest rate $R_{t+1}$. This, in turn, implies that the money growth rate $\tau_t$

$$M_{t+1} = M_t \cdot (1 + \tau_t)$$

is endogenous to money demand at the given level of $R_{t+1}$.

We consider that the central bank can commit to an interest rate rule that assures the achievement of a certain exogenous inflation target. As commitment

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8 New Keynesian models typically assume logarithmic preferences over real money balances (Corsetti and Pesenti, 2001). Here we assume $\varepsilon > 1$, which is sufficient to ensure that the marginal cost to inflating is positive in discretionary inflation and that the solution to the monetary authority problem in the game with the wage setters is always a global maximum (see Neiss, 1999: 361, 368).

9 Capital is assumed fixed and normalized to unity.
is on the interest rate, and monetary growth is determined endogenously by the interest rate policies, the central bank ultimately commits to a fixed money growth rate.

By solving the above model under flexible price it is easy to derive the standard result of a distorted equilibrium:

\[ l_t = \frac{\alpha}{\eta \mu^p \mu^w} \]  \hspace{1cm} (6)

where are the price and wage markups, and the optimality of the Friedman rule, i.e. a negative inflation rate equal to \( \beta - 1 \). By contrast, assuming preset prices and/or wages, employment (or real activity) is unaffected and a Barro-Gordon’s (1983) bias emerges (see Neiss, 1999).

3 Wage moderation and inflation targeting

We assume that central bank can commit to an interest rate rule that assures the achievement of a certain exogenous inflation target. \( m \), before than the wages are set (i.e. preset wages as prices in Neiss, 1999). As commitment is on the interest rate, and monetary growth is determined endogenously by the interest rate policies, the central bank ultimately commits to a fixed money growth rate. Formally, this commitment requires that the central bank implements a constant nominal interest rate rule \( R_t = 1 + \frac{m}{\beta} \), which implies that \( \frac{M_{t+1}}{M_t} = 1 + m \).\(^{10}\) The model is solved by backward induction. The timing of the game is thus as follows. Before the price level is known, at the beginning of the period, the central bank chooses the inflation targeting. Then wage setters must choose the nominal wage rate, \( w_t = \bar{w}_t P_t \), where \( P_t \) is the rational expectation of the price level and \( \bar{w}_t \) is the desired real wage rate. Finally, full price flexibility ensures that markets clear. The model is solved by backward induction. We follow Neiss’ solution method.\(^{11}\)

By solving the representative household’s \((i)\) problem standard first order conditions for consumption are obtained:\(^{12}\)

\[ c_t(j) = C_t \left( \frac{p_t(j)}{P_t} \right) \frac{\pi_t}{\rho - 1} \]  \hspace{1cm} (7)

\[ C_t = \frac{1 + \pi_t C^\pi_{t+1}}{\beta R_{t+1} C^\pi_{t+1}} \]  \hspace{1cm} (8)

where \( \pi_{t+1} = \frac{P_{t+1}}{P_t} - 1 \) denotes the inflation rate.

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\(^{10}\)It is worth noticing that here we define inflation targeting as the case where target variables are variables appearing in reaction functions or behavioral rule (as e.g. Bernanke and Woodford, 1997); while, in the literature is also often referred to variables in loss functions (Svensson, 1997).

\(^{11}\)In this model there is no state variable to link periods and the policy problem is time invariant; see Neiss (1999) for a discussion.

\(^{12}\)Index \( i \) is dropped for simplicity.
The money demand equation is
\[
\frac{M_{t+1}}{P_t} = (1 + \pi_{t+1}) \frac{1}{1-R_{t+1}} \left( \frac{\gamma \beta C_t}{1-R_{t+1}} \right)^{\frac{1}{\gamma}}
\] (9)

Agent faces a trade-off between \(t\) period consumption and \(t+1\) period holdings of nominal money balances. Observe that (9) can also be interpreted as a demand function: when the central bank increases next period nominal money balances, \(coeteris\ paribus\) current consumption increases.

The condition about the optimal labor supply will be introduced at a later stage, following a discussion of firms labour demand.

In the goods market, for any given level of its labor demand, \(l_{t,j}\), the firm must decide the optimal allocation across labor inputs, subject to aggregation technology (4). Firm \((j)\) demand for labor type \((i)\) is
\[
l_{t,j} (i) = \left( \frac{w_t (i)}{w_t} \right)^{-\sigma} l_{t,j}
\] (10)

where \(w_t = \left[ \int_0^1 w_t (i)^{1-\sigma} \, di \right]^{\frac{1}{1-\sigma}}\) is the wage index.

Prices are set as a mark-up over marginal costs:
\[
p_{t,j} = \mu^p \frac{1}{\alpha} w_{t,l_{t,j}}^{1-\alpha}
\] (11)

where \(\mu^p = \frac{1}{\rho}\) denotes the price markup and \(v_t\) is a production subsidy. Labour demand from firm \(j\) is
\[
l_{t,j} = \left( \frac{\mu^p w_t}{\alpha p_{t,j}} \right)^{-\frac{1}{\alpha}}
\]

Aggregating across firms we obtain
\[
Y_t = \lambda l_t \alpha
\] (12)

\[
l_t (i) = \left( \frac{w_t (i)}{w_t} \right)^{-\sigma} l_t
\] (13)

\[
l_t = \left( \frac{\mu^p w_t}{\alpha p_t} \right)^{-\frac{1}{\alpha}}
\] (14)

As nominal wages are predetermined to prices, the wage-setting problem is solved by choosing the nominal wage that maximizes the expected value of (1) subject to the rational expectation of the price level, \(p_t^e\). In this case the wage setting rule takes this form:
\[
w_t = \eta \mu^C_p p_t^e
\] (15)

and the ex-ante wage markup, \(\mu\), becomes policy endogenous.
The monopolistic wage setters in fact anticipate that in equilibrium individual real money holdings fall with consumption:13

\[ \frac{M_{t,i}}{P_t} = \left( \frac{\gamma \beta C_{t,i}^e}{1 + m - \beta} \right)^{\frac{1}{\varepsilon}} \] (16)

and that consumption is a negative function of the real wage. Thus, under an inflation targeting rule, wage setters anticipate that real money balances will fall due to the adverse effect of the wage choice on consumption.

Formally, the wage setters maximize (1) subject to (9), which, by imposing rational expectations (\( \pi^e = m \) and \( P_t^e = P_t \)), becomes (16). The wage setters’ first order condition in the symmetric equilibrium is:

\[ \frac{\sigma - 1}{\sigma} (1 + \delta_m) = \frac{\nu}{w} C_t^{e} \] (17)

where the adverse effect is measured by \( \delta_m = \frac{1}{\varepsilon} \left[ \gamma \left( \frac{1 + m - \beta}{\beta} \right)^{\varepsilon - 1} C_t^{e \varepsilon} \right]^{\frac{1}{\varepsilon}} \), which is increasing in \( m \) (recall that \( \varepsilon > 1 \) for the marginal cost of inflation to be positive.)

Straightforward manipulations lead to equation (18).

\[ w_t = \eta \mu w \left( 1 + \delta_m C_t^{e} P_t^{e} \right) \] (18)

where:

\[ \delta_m = \frac{1}{\varepsilon} \left[ \gamma \left( \frac{1 + m - \beta}{\beta C_t^{e}} \right)^{\varepsilon - 1} \right]^{\frac{1}{\varepsilon}} \] (19)

For any level of expected consumption the ex-ante wage mark-up, \( \mu \), is disciplined by the inflation target, and the strength of this effect is growing with \( m \). Note also that if the central bank targeting is inspired by the Friedman argument \( \delta_m \) is zero, otherwise it is positive.14

By combining we obtain an implicit form for employment

\[ l_t = \frac{\alpha 1 + \delta_m}{\eta \mu \nu^{\varepsilon} \mu^{w}} \] (20)

recall that the endogenous markup is a function of the consumption level, which is non linearly related to the labor supply by the real wage.

By differentiating equation (20) we obtain:

\[ \frac{\partial l_t}{\partial m} = \frac{\delta_m (\varepsilon - 1) l_t}{(1 + m - \beta) [(1 + \delta_m) \varepsilon + \alpha \delta_m (\varepsilon - 1)]} > 0 \] (21)

The effect of the target on employment (and thus consumption) is positive.

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13 Condition (16) is obtained from (9) imposing rational expectations.
14 Of course, we abstract from the case where the central bank creates unnecessary deflation.
The rationale for the above result can be explained by inspecting equation (16). Wage setters anticipate that, given the target $m$, wage moderation increases real money balances. Such an effect is stronger the higher the target. In other words, under flexible wages, the trade union optimization problem is solved by choosing a real wage such that consumption falls below the perfectly competitive rate, i.e. $\mu^w$. This loss of utility is more than compensated for by the corresponding reduction in labour effort. By contrast, when wages are predetermined, unions will anticipate the effect of the target on the correlation between lower consumption and real money balances.

4 Welfare analysis

By targeting the Friedman rate the flexible price equilibrium is obtained. A rise in the target clearly implies a trade off between inflation and real output as in Benigno and Ricci (2008). The effects of targeting induced trade-off on welfare can be evaluated by using equation (1). Intuitively, the optimal inflation target will be between the Friedman deflationary rule and the positive inflation rate ensuring the achievement of the Pareto optimal employment, according to the agents’ preferences. However, due to the complexity of the analytical solution of the model, we must rely on numerical simulations to evaluate it.

We calibrate the model by setting the discount rate ($\beta$) at 0.99, which corresponds to a yearly long-term real interest rate of 3%, the labor coefficient ($\alpha$) at 0.6 and we endogenously determine the scale parameter of labor disutility ($\eta$) to normalize the Pareto optimal level to 1. We assume the income elasticity of money demand ($1/\varepsilon$) to be 0.516 and set the money scale parameter ($\gamma$) to 0.005. Thus, to check the result robustness, we also consider alternative parameterization. We fix the price and wage markups to 1.1 and 1.2 respectively (i.e. $\rho = 0.9$ and $\sigma = 6$).17

Welfare under different targets is plotted in figure 2.18

15 The endogenous markup is a function of the consumption level, which is non linearly related to both the subsidy and the real wage by the labor supply.
16 See e.g. Choi and Oh (2003), Dib (2004), Knell and Stix (2005) and references therein.
17 We also test the robustness of our results by considering two alternative scenarios where elasticity of substitution ($\sigma$) is chosen to obtain different wage markups: 1.05 and 1.15. The former is closer to the calibration for the United States of Christiano et al. (2005) whereas the latter to that of Gali et al. (2007). Results are almost identical.
18 In the figure inflation targets are annualized.
In our quite standard calibration, the optimal target is about 2%.\footnote{Results are robust with respect to reasonable changes in the parameters. Additional simulations are available upon request.}

Regarding the effect of the money scale parameter, we consider four scenarios to which are associated the following values for $\gamma$: 0.0005 (panel (a)), 0.005 (panel (b)), 0.05 (panel (c)), 0.5 (panel (d)). The scenarios are plotted in the following figure.

Figure 2 – Welfare evaluation
The figure confirms the robustness of our results and shows that an increase in agents’ concerns for the real money balances implies a greater optimal target. It is worth noticing that, even if the money scale parameter is very low, deviations from the Friedman rule are desirable.

5 Conclusions

Our model provides a rationale for targeting a positive inflation rate in the long run. We thus suggest a reconsideration of inflationary equilibria as properly designed monetary policies can take advantage of predetermined nominal wages to discipline monopolistic wage setters, this requires accepting a non-zero inflation. A commitment to a non-zero inflation target in fact implies that the central bank increases the expected marginal benefit of holding money. Thus it induces wage setters to stimulate consumption by supporting employment through moderate wage claims.

The key assumption for our result is that, differently from standard New Keynesian models usually based on flexible wages, all nominal wages are pre-
determined to both the individual households’ and the policymaker’s decisions. As a consequence, even if atomistic labor markets are considered (as usual in the New Keynesian literature),\(^{20}\) the wage setters internalize the effects of their choice on money holdings for the representative household’s welfare.

References


