UNIVERSITA' DEGLI STUDI DI ROMA "LA SAPIENZA"
DIPARTIMENTO DI STUDI GEOECONOMICI, LINGUISTICI, STATISTICI, STORICI
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Policy and Controllability under Rational Expectations[*]

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Working Paper n°52
Gennaio 2009

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[*] The authors are grateful to R. Neck for useful discussions and comments on an earlier draft. Nicola Acocella and Giovanni Di Bartolomeo thank the University of Rome 'La Sapienza' for funding. Giovanni Di Bartolomeo also acknowledges the financial support of the European Union (Marie Curie ToK, contract No 014288, MTKD-CT-2004-014288) and the University of Crete for their hospitality.

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JEL Classification: C61, C62, E52, E61, E62.

Keywords: rational expectations, controllability, stabilisability, policy neutrality, policy design.

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1. Introduction

Policymakers routinely use announcements as policy instruments to influence future expectations. The well known announcement or signal effect, in fact, implies that the announcement of a change in policy will affect agents' behaviour, even before the change is actually put into effect. Rational policymakers should thus internalize announcement effects and use signals strategically. The economic policy literature however does not have a formal model of whether, and under what conditions, policy announcements can affect economic performance. But the conventional wisdom is that policy announcements are likely to prove ineffective or inconsistent with private expectations. That in turn is inconsistent with the very evident attempts by governments and central banks to manage expectations. Is it then

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reasonable that they should try?

Although announcements are often used strategically by policymakers in many areas,[1] the work of Barro (1974), Sargent and Wallace (1975) and Lucas (1976) has led many people to regard policy announcements, and commitments to achieve certain policy targets, with suspicion in a world with forward-looking expectations. Time inconsistency and rational expectations are said to imply that such commitments cannot be considered credible and would inevitably lead to Pareto inferior outcomes. However, the credibility problem can be solved in a repeated game context (Kreps and Wilson, 1982; Milgrom and Roberts, 1982). In that context, there would be no need for the private sector to adjust their expectations of the outcomes as a result of the government's interventions. And for reputation, the policymaker can mimic a *forthright type* who always honours policy announcements. In this context, the final few periods of the game apart, policy signals are always credible. Applications have been made to a vast array of credibility problems.

In this paper we approach the problem from a fresh perspective, by considering endogenous or rational expectations directly in a traditional Tinbergen framework and showing that under certain circumstances the usual dichotomy between rational expectations, on the one hand, and the ineffectiveness or time inconsistency of policy actions on the other, may not arise. We show that, if expectations are rational, policy invariance and time inconsistency emerge *only* in the special case of a Tinbergen flexible targets problem. In the more general case – that is in an unconstrained optimisation (free from externally imposed preferences or optimisation techniques) and where policymakers can be said to control the economic system either statically or dynamically – the endogenization of expectations will not only present the policymakers with no problem of how to set their policies consistently; but may actually add to the scope of their policy instruments, in effect giving them additional sources of effective policy power. Our contribution is therefore to bridge the gap between the practice and what our traditional policy models tell us.

This essay is one in a line of papers rehabilitating the theory of economic policy, extended so far to multiple policymakers and strategic policy games.[2] In this paper we add rational expectations to the classical theory in a single policymaker context and derive the conditions for both static and dynamic controllability for that case. It is organized as follows. Section 2 presents the reduced and final form of a model with a single policymaker and

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[1] See Persson and Tabellini (1990) for a general survey; and Blinder *et al*. (2008), Woodford (2005), or Rudebusch and Williams (2008) for examples in central banking. Instances in the financial markets will be found in Balduzzi *et al*. (2001), Andersen *et al*. (2003), Fair (2003) and Faust *et al*. (2007).

[2] See Acocella and Di Bartolomeo (2006), Acocella *et al*. (2006, 2007), Di Bartolomeo *et al.* (2008).

rational expectations. In section 3 we deal with the conditions for static and dynamic controllability of this model and demonstrate that dynamic controllability can be enhanced by rational expectations. We also derive the rational expectations version of Wonham's (1974) stabilisability theorem. Section 4 concludes with some implications for policy design.

2. The economic model with a single policymaker

Without loss of generality, we can write the generic linear rational expectations model, in its reduced form, as follows:

$$(1) \qquad y_t = Ay_{t-1} + By_{t+1/t} + Cx_t + v_t \qquad\qquad \text{for} \quad t = 1.....T$$

where T is a finite, but possibly large number; and where $y_{t+1/t} = E(y_{t+1}/\Omega_t)$ denotes the mathematical expectation of y_{t+1} conditional on Ω_t (the information set available at *t*). In this set up, y_t is a vector of *n* endogenous variables at time *t*; x_t is a vector of *m* potential policy instruments; and v_t is a vector of exogenous shocks and/or other influences which have a known mean, but come from an unspecified probability distribution. We assume, as is conventional in this literature, that none of the endogenous variables in y_t contains a unit root. Likewise, the matrices *A*, *B* and *C* are constant and of order *n*, *n*, and $n \times m$ respectively, and have at least some elements which are non-zero.

This model can now be solved from the perspective of any particular period, say *t*=1, by putting it into final form conditional on the information set available in that period:[3]

$$(2) \quad \begin{pmatrix} y_{1/1} \\ : \\ : \\ : \\ y_{T/1} \end{pmatrix} = \begin{bmatrix} I & -B & 0 & . & 0 \\ -A & I & & . & . \\ 0 & & . & & 0 \\ . & . & & . & -B \\ 0 & . & 0 & -A & I \end{bmatrix}^{-1} \left\{ \begin{bmatrix} C & 0 & . & . & 0 \\ 0 & . & & & . \\ . & & . & & . \\ . & & & . & 0 \\ 0 & . & . & 0 & C \end{bmatrix} \begin{pmatrix} x_{1/1} \\ . \\ . \\ . \\ x_{T/1} \end{pmatrix} + \begin{pmatrix} v_{1/1} \\ . \\ . \\ . \\ v_{T/1} \end{pmatrix} + \begin{pmatrix} Ay_0 \\ 0 \\ . \\ . \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ . \\ . \\ 0 \\ By_{T+1/1} \end{pmatrix} \right\}$$

In this representation, y_0 is a known initial condition at $t = 1$; and $y_{T+1/1}$ is an assumed or projected terminal condition – most likely one that describes the economy's expected long run equilibrium state as part of Ω_1. Although (2) has been solved from the point of view of Ω_1, it must be understood that it could have been derived for each Ω_t, t = 1....T, in turn where $y_{j/t} =$

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[3] Hughes Hallett and Fisher (1988), Hughes Hallett *et al.* (1996).

$E_t(y_j)$ if $j \geq t$, but $y_{j/t} = y_j$ if $j < t$; and similarly for x and v. But for simplicity, we will consider the Ω_1 case only in what follows. The generalisation to any other value of t is then obvious.

Second, the equation to which (2) is the solution makes it clear that neither the policymakers, nor the private sector are required to move off their expected paths (make expectational errors) for the policies to work. In fact equation (2), also (4) below, show just the opposite; those expectations are exactly consistent with what private agents and policymakers expect the policy outcomes to be. The only question is whether policies, or announcements of policies, can be found that would shift the expectations path itself by the required amount. The task of this paper is to find the conditions under which this can be done. Hence our purpose is to determine when it is possible to shift expectations in such a way that the economy's final outcomes can reach certain target values at specified points of time; *and* when it is not possible.

The extension of (1), and hence (2) to allow any number of leads and lags is detailed in an appendix to this paper. In addition, it is easy to show that this final form solution generally exists, given (1), since the inverse matrix in (2) is always well defined. To see this, define the Toeplitz matrix itself to be: $T_T = \begin{bmatrix} I & -B & 0 & . & 0 \\ -A & I & & . & . \\ 0 & & . & & 0 \\ . & . & & . & -B \\ 0 & . & 0 & -A & I \end{bmatrix}$.

This matrix is of order nT. Using the partitioning by time, the determinant of T_T is

$$|T_{T-1}|.|I_n - (-B, 0.....0)T_{T-1}^{-1}(-A', 0.....0)'|$$

However $|T_{T-1}| = |T_{T-2}||I_n - (-B, 0.....0)T_{T-2}^{-1}(-A', 0.....0)'|$, and so on. Hence we can write (3)

$$|T_i| = |T_{i-1}||I_n - (-B : O)T_{i-1}^{-1}(-A' : 0)'| = |T_{i-1}||I_n - BA| \neq 0 \qquad \textit{for } i = 2\ldots\ldots T$$

The equalities in (3) follow from the partitioning in T_{i-1}^{-1} and repeated applications of the Woodbury formula for the inverse of a matrix sum; and the inequality from the absence of unit roots in (1) or AB.[4] But $|T_1| = |I_n|$. Hence the inverse always exists by induction.

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[4] A weaker condition, if $T \rightarrow \infty$, would be the usual saddle point property (Hughes Hallett and Fisher, 1988). Notice that this result automatically implies that the traditional vertical Phillips curve model would not be controllable in the long run since T_T would be lower triangular with A having a unit root (if A in (1) is associated with contemporaneous expectations, or lagged values as an approximation). It was our purpose to identify conditions when the system is not controllable, as well as when it is. The unit root condition on A (or AB in our more general formulation) is one; the other, a failure of the rank condition in proposition 3.

Given that (2) always exists, we can now rewrite the final form model in the following way:

$$\begin{pmatrix} y_{1/1} \\ . \\ . \\ . \\ y_{T/1} \end{pmatrix} = \begin{bmatrix} R_{11} & . & . & . & R_{1T} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ R_{T1} & . & . & . & R_{TT} \end{bmatrix} \begin{pmatrix} x_{1/1} \\ . \\ . \\ . \\ x_{T/1} \end{pmatrix} + \begin{pmatrix} b_{1/1} \\ . \\ . \\ . \\ b_{T/1} \end{pmatrix}, \text{ or}$$

$$(4) \qquad y = Rx + b$$

where $R = T_T^{-1}(I \otimes C)$, $b = T_T^{-1}\{E(v/\Omega_1) + (A':0)'y_0 + (0:B')'y_{T+1/1}\}$, and "$\otimes$" denotes a Kronecker product. In this formulation, each $R_{t,j} = \partial y_{t/1} / \partial x_{j/1}$ defines an $n \times m$ matrix of policy multipliers for $t, j = 1 \ldots T$. Notice also that R is not block triangular: so $R_{t,j} \neq 0$, even if $t < j$. In other words, equation (4) implies $R_{t,j}$ is a matrix of conventional policy multipliers between $y_{t/1}$ and $x_{j/1}$, with a delay of $t - j$ periods between implementation and realization if $t > j$. By contrast, $R_{t,j}$ represents a matrix of anticipatory effects on $y_{t/1}$, of an announced or anticipated policy change $x_{j/1}$ at some point in the future, if $t < j$.[5]

3. Controllability with forward looking behaviour

3.1 Static controllability

Static (or Tinbergen) controllability defines the set of conditions which must hold if an arbitrary set of target values can be achieved for the endogenous variables y_t in each period – at least in expectation, given that the original model is stochastic. Define those target values to be $y_{t/1}^d$, where superscript d denotes a desired value from the perspective of period 1. We then define y^d to be a stacked vector of those desired values across time periods.

Static controllability, in each period in turn, evidently requires the matrix R in (4) to possess an inverse:

$$(5) \qquad x = R^{-1}(y^d - b)$$

where y, x and b are all understood to be expectations conditioned on the current information set Ω_t, for each $t = 1 \ldots T$, as noted at (2) and (4). Hence:

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[5] A conventional "backwards looking" model will have $R_{t,j} = 0$ for all $t < j$; *and* constant multipliers $R_{t,j} = R_{t-j}$ for $t - j = 0.....T-1$, if the model at (1) is linear. Neither of these things is true in (4).

Proposition 1:

Static controllability under rational expectations, as in a conventional backwards looking model, requires the model to have as many independent policy instruments as target variables in each time period. Hence there is no generalization or change in the static controllability conditions when there are rational or forward looking expectations.

Proof: From (4), $R = T_T^{-1} C_T$ where $C_T = I_T \otimes C$. Hence $R_T^{-1} = (T_T^{-1} C_T)^{-1} = C_T^{-1} T_T$ exists if and only if $C_T^{-1} = I_T \otimes C^{-1}$ exists, since we already know that T_T^{-1} exists. But the instru-ment coefficient matrix, *C*, can only possess an inverse if $n = m$ and C has full rank. Those are also the conditions which provide period-by-period static controllability in a backward looking model, whether static ($A = 0$, $B = 0$) *or* dynamic ($B = 0$).■

Comments:

i) As always $n = m$, well-known as the Tinbergen theorem, is a necessary condition for static controllability; linear independence in the impacts of the instruments on the targets (together with $n = m$) is sufficient. This corresponds to the conventional case studied in Hughes Hallett (1989).

ii) If the same number of target and instrument variables appears in each time period, then the necessary condition emerges directly from the matrix inverse in (5) since *R* is of order $nT \times mT$. However, if they differ across time periods, then we need $n_t = m_t$ in each period if static controllability is to hold across the whole policy period since $I \otimes C^{-1}$ now becomes $C_T^{-1} = diag\{C_t^{-1}\}$. The sufficient condition, in terms of linear independence within C_T, cannot be seen from *R* in this case.

iii) In the event that we have surplus instruments, $m > n$, then we may transfer $m - n$ of them from $x_{t/1}$, times their coefficients from *C*, to the corresponding element of $v_{t/1}$ in (2) before proceeding with proposition 1 on the reduced system.

3.2 Dynamic controllability

Conventionally an economy (model) is said to be dynamically controllable if a sequence of instrument values $x_1, \ldots\ldots, x_t$ can be found to reach any arbitrary values, y_t^d, for the target variables in period *t* (at least in expectation), given an arbitrary starting point y_0. In that case, we are no longer concerned with the period-by-period controllability of the target variables between periods 1 and $t - 1$. Viewed from period 1, dynamic controllability therefore requires a sequence of intended instrument values $x_{1/1},.....,x_{T/1}$ that guarantee that $y^d_{t/1}$ is reached in period t. Given (4), this will be possible only if the sequence of policy multipliers and anticipatory effects, $R_{t,1}.....R_{t,T}$, is of full rank: $r(R_{t,1}....R_{t,T}) = n$, given an arbitrary initial state y_0 and a specific terminal condition $y_{T+1/1}$.

Proposition 2:

The economy represented by (1) is dynamically controllable over the interval (1, *t*), when $T \geq n$, if $r(R_{t,1}....R_{t,n}) = n.$[6]

Proof: $y^d_{t/1} = (R_{t,1}....R_{t/T})x + b_{t/1}$ is reachable over (1, *t*), using a Moore-Penrose generalized left inverse, if $r(R_{t,1}....R_{t,T}) = n.$ But if $T \geq n$, then $r(R_{t,1}...R_{t,T}) =$ $r(R_{t,1}...R_{t,n}) = n$ which provides the result.■

Comments:

i) Proposition 2 is an interesting and important generalization over the conventional case with backwards looking models. If $n > t$, which is entirely possible for small values of *t*, dynamic controllability will be available through the reactions of $y_{t/1}$ to the implemented policy choices $x_{1/1}...x_{t/1}$; ***and*** through the anticipatory effects of announced or anticipated policy interventions that still lie in the future, $x_{t+1/1}....x_{n/1}$. In other words, the policy maker can use policy announcements, in addition to actual interventions, to guide the course of the economy. In a conventional model that would not be possible since $R_{t,j} = 0$ for all $j > t$. In effect, the policymaker now has a greater number of policy "instruments" at his disposal than in an economy without anticipations.

ii) Corollary: all $y_{t/1}$, including the targets of the first period $y_{1/1}$, are now dynamically controllable if the rank condition in proposition 2 holds. That too is an important extension over the conventional case where period $t = n$ is the earliest date at which we can guarantee controllability if there is a single policy instrument; or $t = n/2$ if there are two instruments, and so on. Here $y_{1/1}$ is controllable from the first period, even if there are insufficient instruments ($m < n$), provided that both $T \geq n$ and proposition 2 holds. The astute policymaker will realise that good communication lies at the heart of the policy problem if he/she wants to reach their policy targets in the early periods or at lower cost, a fact which has already attracted

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[6] This proposition provides a sufficient condition for dynamic controllability. The corresponding necessary condition involves a smaller subset of R_{tj} having full rank depending on how many policy instruments are available (see section 3.4). Proposition 2 is therefore given for the general case with $m \geq 1$.

considerable interest in central banking circles (Woodford 2005, Blinder et al, 2008, Rudebusch and Williams 2008).

iii) Evidently dynamic controllability is also possible with a much reduced instrument set, compared to static controllability. There are two parts to this reduction: a) the ability to use one or more instruments repeatedly rather than a group of several instruments used once and in parallel; and b) the ability to augment, or even replace, parts of an existing instrument set with announcements of future policy changes.

iv) There is also a distinction in that the $x_{1/1}...x_{t/1}$ values will be implemented decisions when it comes to the controllability of $y_{t/1}$; but that the $x_{t+1/1}....x_{n/1}$ values, being policy announcements, may never actually be carried out. However, because they lie in the future from the perspective of $y_{t/1}$, any subsequent time inconsistency plays no role in the controllability of $y_{t/1}$ as long as they are *genuinely* held expectations at that point.

v) For that reason, we have taken the first *n* multiplier matrices for the rank condition in proposition 2. That is an arbitrary choice; we could have taken any *n* sub-matrices from $R_{t,1}.....R_{t,T}$. But a choice of the first *n* maximizes the proportion which represents actual policy choices as opposed to potentially fungible policy announcements.

Proposition 3:

Forward looking or rational expectations enhance the power to control an economy over time in that: a) policy announcements may be used to supplement and extend the impact of conventional policy instruments; and b) controllability is now available, with a reduced instrument set, from much earlier; and from $t = 1$ if $r(R_{11}...R_{1n}) = n$.

Proof: comments i), ii) and iii) above, and proposition 2.■

3.3 Stabilisability under rational expectations

We can apply the reasoning underlying propositions 2 and 3 to show that any economy can also be stabilized to an arbitrary degree under rational, forward looking expectations if it is dynamically controllable in the sense of proposition 2. An arbitrary degree of stabilisation means that policy rules can be found to make the economy follow an arbitrarily stable path, based on an arbitrary set of eigenvalues. This is the rational expectations extension of the standard stabilisability theorem for backward looking or physical systems.[7]

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[7] Wonham (1974).

Proposition 4:

For any economy represented by (1), with arbitrary matrices *A*, *B* and *C*, we can always find a set of policy rules, $x_{t/1} = K_t y_{t-1/1}$, such that the controlled economy is stabilisable (up to an arbitrary set of eigenvalues) if the economy itself is controllable in the sense of proposition 2.

Proof: Equation (1), with arbitrary coefficient matrices *A*, *B* and *C*, can be reduced to its final form (2). Substituting the policy rule $x_{t/1} = K_t y_{t-1/1}$ for each t = 1….T shows that the controlled economy will behave as, similarly to (4),

$$(6) \qquad \begin{pmatrix} y_{1/1} \\ \cdot \\ \cdot \\ \cdot \\ y_{T/1} \end{pmatrix} = \begin{bmatrix} R_{11} & \cdot & \cdot & \cdot & R_{1T} \\ \cdot & \cdot & & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ R_{T1} & \cdot & \cdot & \cdot & R_{TT} \end{bmatrix} \begin{bmatrix} K_1 & 0 & \cdot & \cdot & 0 \\ 0 & \cdot & & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & 0 \\ 0 & \cdot & \cdot & 0 & K_T \end{bmatrix} \begin{pmatrix} y_0 \\ y_{1/1} \\ \cdot \\ \cdot \\ y_{T-1/1} \end{pmatrix} + \begin{pmatrix} b_{1/1} \\ \cdot \\ \cdot \\ \cdot \\ b_{T/1} \end{pmatrix}$$

where $y_{0/1} = y_0$. Thus

$$(7) \qquad y_{t/1} = (R_{t,t-n}....R_{t,t}) \begin{pmatrix} K_{t-n} & 0 & 0 \\ 0 & \cdot & 0 \\ 0 & 0 & K_t \end{pmatrix} \begin{pmatrix} y_{t-n-1/1} \\ \cdot \\ y_{t-1/1} \end{pmatrix} + \sum_{j=1}^{t-n-1} R_{t,j} K_j y_{j-1/1} + \sum_{j=t}^{T} R_{t,j} K_j y_{j-1/1} + b_{t/1}.$$

For an economy to be stabilisable at *t*, it must possess the property that it would return to the initially expected path, whatever the initial conditions and shocks experienced up to that point, if no further shocks or changes in expectations emerge (Wonham, 1974). That property clearly exists if the iteration matrix $(R_{t,t-n}....R_{t,t}) \begin{pmatrix} K_{t-n} & 0 & 0 \\ 0 & \cdot & 0 \\ 0 & 0 & K_t \end{pmatrix}$ has roots in the unit circle.

But $y_{t/1}$ will follow any arbitrarily stable path if we can pick $K_{t-n}.....K_t$ so as to imply an arbitrary set of eigenvalues for that matrix. Suppose we choose the iteration matrix $D = S\Lambda S^{-1}$ where Λ is a diagonal matrix of the chosen eigenvalues. Then, so long as $t > n$ and $r(R_{t,t-n}...R_{t,t}) = n$ hold, we can calculate the required $K_{t-n}.....K_t$ from the generalized inverse $(R_{t,t-n}...R_{t,t})^{+} D$ and apply a block diagonalisation to the result.[8]■

Comments:

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[8] A block diagonalisation exists since $(R_{t,t-n}....R_{t,t})^{+}D$ is square, and the Jordan canonical form exists.

i) Policies that imply stabilisability are obviously not unique, even if the choice of Λ is unique. We could have used policies further in the past, or policy announcements about the future, from the second and third terms in (7) to derive proposition 4 (given $T \geq n$). In any case, the generalised inverse is not unique.

ii) As a result, we can infer that a rational expectations model that is dynamically control-able at $t = 1$, as in proposition 2, is also stabilisable from $t = 1$. In that sense proposition 4 generalizes on Wonham's original theorem where stabilisability is achieved for the first time in period n.

3.4 An extra generalization

If only a subset of the variables in each vector y_t are genuine targets of policy (say s of them), then we may delete the $n - s$ rows from each y_t, and from the blocks of policy multipliers $R_{t,1} \ldots . R_{t.T}$ that correspond to the non-target variables, before moving on to evaluate our controllability conditions.

The **static controllability condition** will now be $s = m$, and that the condensed R matrix just constructed should have rank sT. But it is no longer possible to characterize the sufficient condition part of the problem in terms of the elements of C (or of T_T and C). Nevertheless it will be easier to control this subset of immediate policy targets, than it is to control the whole economy.

For **dynamic controllability**, we follow the same logic and apply proposition 2 to the condensed system. We get, as a sufficient condition, $r(R_{t,1} \ldots .. R_{t,s}) = s$ for controllability over the interval [1, t] – implying dynamic controllability from the first period as before. Once again, it is easier to control the subset of target variables than to control the entire economy by period t.

4. Concluding remarks.

This paper was set up to determine the conditions under which policies can still affect the outcomes, and hence private sector expectations, when economic performance and policy choices are influenced by rationally chosen forward looking expectations; and, equally, to determine the conditions under which the outcomes cannot be controlled, or private sector expectations managed, so as to achieve certain objectives. Our key result is that policy targets can be controlled, and expectations managed through the policy process, as long as the forward dynamics (from private expectations of future outcomes) and the backward dynamics (from past policies/outcomes) do not interact to imply a unit root. If no unit root is implied, then expectations can be managed to increase controllability in the conventional sense and also to make policies more effective from an earlier date. But if there is a unit root, it means that the forward and backward dynamics exactly offset the status quo – so that the anticipated and feedback effects of a policy change exactly cancel out in their impact on current outcomes. Therefore, policies can have no effect.

This explains our results: the unit root comes from the matrix product AB in equation (1), where A controls the effect of past decisions and outcomes as chosen by the policy makers, and B the effect of expected future outcomes as anticipated by the private sector. Only where there is a conflict between the two, such that the influence of the expectations exactly offsets the planned changes to the existing state, will policy become ineffective. This invariance result should therefore be attributed to a conflict between agents and policymakers; a conflict which may take place if the private sector can set their expect-ations to have a *sufficient* effect on the policymakers' targets (since there is obviously no incentive to align expectations with targets over which the authorities have no control), rather than to myopia or ill-intentions by the latter. But that has to be a special case. Otherwise, if the authorities satisfy the rank condition in proposition 3 (so that they do have controllability in the usual sense), expectations can be managed: that is, brought into line to assist with what the policymakers intend to achieve.

Thus, although we have shown that rational expectations can make it impossible for the policymaker to affect the outcomes in certain economic models due to policy invariance, or time inconsistency, this is not a general result. In fact it will only happen if the matrix product AB has a unit root, or if the rank condition in proposition 3 fails.[9] Both conditions depend on particular parameter values; so neither will hold in general. Hence rational expectations do not, in themselves, prevent controllability in either its static or its dynamic form. On the contrary, they will typically enhance the effectiveness of economic policy. However, there are two special cases where rational expectations do create policy invariance: the long run Phillips curve and New Keynesian models without persistence (in both cases where inflation reacts one-for-one to changes in expected inflation, as is conventionally assumed). In the

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[9] There could still be particular values of A and C such that the introduction of $B \neq 0$ makes $(R_{t,1} \ldots. R_{t,n})$ have less than full rank when, with $B=0$, $(R_{t,1} \ldots. R_{t,n})$ had had full rank. Although this could happen by chance with certain parameter values, it cannot be a general outcome. In any case, in that situation there would be a problem of the non-existence of the underlying equilibrium.

former case, $B = 0$ and A is diagonal with a unit element in one position.[10] Although the inverse in (2) still exists, it is a companion matrix and the border elements do not decay as $T \to \infty$. Consequently the policymakers cannot have controllability in the long run, though they may well have it in the short to medium term. The same happens in the New Keynesian model; now $A = 0$, and B is diagonal with a unit element.[11] Again, and for the same reason, policymakers will not have controllability as $T \to \infty$ although they may have it for shorter horizons.[12]

The implications of our results are important. All dynamic problems that imply the achievement of a given target at a certain moment of time – such as fiscal consolidation, or the achievement of a set of macroeconomic targets as a pre-requisite to creating a currency union – will find an important ally in the existence of rational expectations if dynamic controllability is satisfied. The policy problem is no longer a matter of how to find a credible commitment, but of when the policy changes should be announced given the economy's lag structure. Similarly, policy announcements become useful instruments for stabilizing an economy hit by temporary shocks since, under either static or dynamic controllability, the stabilisability property in backward looking models generalizes to forward looking models *and* allows a sequence of stabilising actions to become effective from an earlier point.

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[10] This reflects the usual assumption that agents expect inflation at its last observed value. If they expect it at the current value, the inverse in (1) becomes block diagonal with $I-A$ on the diagonal which implies singularity. That means controllability would be lost in the short term too.

[11] This will also happen with full persistence if either fiscal policy or monetary policy is absent or unused.

[12] Some specific applications in a New Keynesian context will be found in Hughes Hallett *et al*. (2008).

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Appendix: A Generalization to Multiple Leads and Lags

We now consider a general linear rational expectations model, with p lags and q lead or expectations terms. This can be converted to a first order model, such as in (1), as follows. The (*p*, *q*) model is of the form:

(A1) $y_t = A(L)y_t + B(L^{-1})y_{t/t} + Cx_t + v_t \qquad t = 1, \ldots, T$

where $A(L) = A_0 + A_1 L + A_2 L^2 + \ldots + A_p L^p$ with $A_0 = 0$;

and $B(L^{-1}) = B_0 + B_1 L^{-1} + \ldots\ldots + B_q L^q$ with B_0 are both polynomials in the lag operator $Ly_t = y_{t-1}$. In such a model, we can rewrite (A1) by stacking the variables as follows:

(A2)
$$\begin{pmatrix} y_{t+q-1} \\ \cdot \\ y_t \\ \cdot \\ y_{t-p+1} \end{pmatrix} = \begin{bmatrix} 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & 0 & \cdot & \cdot & 0 \\ \cdot & A_1 & \cdot & \cdot & A_p \\ \cdot & I & 0 & \cdot & 0 \\ 0 & \cdot & 0 & I & 0 \end{bmatrix} \begin{pmatrix} y_{t+q-2} \\ \cdot \\ y_{t-1} \\ \cdot \\ y_{t-p} \end{pmatrix} + \begin{bmatrix} 0 & I & 0 & \cdot & 0 \\ \cdot & \cdot & 0 & I & 0 \\ B_q & \cdot & \cdot & B_1 & 0 \\ 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix} \begin{pmatrix} y_{t+q/t} \\ \cdot \\ y_{t+1/t} \\ \cdot \\ y_{t-p+2} \end{pmatrix} + Cx_t + v_t$$

or, in obvious notation,

(A3) $\tilde{y}_t = \tilde{A}\tilde{y}_{t-1} + \tilde{B}y_{t+1/t} + Cx_t + v_t$

which is in exactly the same form as the model set out in (1) of the main text.